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Individual Choices
in a Non-Consequentialist Framework:
A Procedural Approach

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Abstract

Non-consequentialist features of a choice procedure may matter. The procedural aspect behind availability of alternatives may induce an individual to abstain from choosing any element at all so that the choice set becomes empty. In this paper, the concept of choice is related to procedural aspects of different kinds. Availability of objects, different aspects of production as such and shrinkages from an originally given set of options to various of its subsets are considered. Rationalizability of choice in such contexts is studied axiomatically. We relate our conditions to the standard consistency conditions from traditional choice theory.

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1 Motivation and Some Examples

It is probably true to say that most of welfare economics is consequentialist in nature. In other words, what really matters in analyses of welfare are outcomes in terms of individual and social welfare. However, as Suzumura and Xu (2001) write, “there do exist people who care not only about welfaristic features of the consequences, but also about non-welfaristic features of the consequences or even non-consequential features of the decision-making procedure through which these consequences are brought about” (p. 424). Within a firm, for example, most employees will probably prefer that major organisational changes be carried out after some general discussion that involves the staff and not at the bidding of the board of directors. In the political sphere, many people will most likely prefer that a new policy be brought about through public debate and not clandestinely.

In bargaining theory and expected utility theory analogous phenomena exist. Let E_1 and E_2 be two economic environments that yield identical utility-possibility sets, i.e. $S(E_1) = S(E_2)$. If f stands for a particular bargaining solution, we obtain $f(S(E_1)) = f(S(E_2))$ as a fundamental principle of bargaining analysis. Both information on how the underlying commodity allocations came about and information on how the set of feasible utility allocations was arrived at, are aspects that do not matter under f . One could, again, argue that these features should matter, at least under certain circumstances. In expected utility theory à la von Neumann–Morgenstern, a one-stage lottery and a multi-stage lottery are judged to be equivalent utility-wise as long as the outcomes and the net cumulative probability of each outcome are the same.

Coming back to bargaining theory, to Nash’s (1950) solution in particular, his condition of independence of irrelevant alternatives requires that if the solution for a certain set of utility allocations S is still possible or feasible for some subset $S' \subseteq S$, then this solution should also resolve the bargaining problem for S' . In other words, this particular shrinkage of the bargaining set itself is of no relevance.

In experimental game theory of ultimatum games, it is often modeled that the responder’s behaviour is only responsive to the division rule proposed by the proposer. The procedure or process from which the proposer chooses the division rule is not relevant to how the responder reacts. There has been, however, an increasing recognition on the part of experimental economists that the responder’s behaviour is different whether the division rule is chosen by the proposer from a larger set of feasible division rules or

the same division rule is chosen by the proposer from a singleton set consisting of just one division rule (the responder has no choice but to choose the given rule).

Traditional choice theory does not consider procedural aspects or the latter are deemed insignificant for choice. Let there be a finite set of alternatives X and strict subsets $A \subset X$. Furthermore, let \mathcal{N} be a finite set of distinct procedures (concerning production of the objects or their availability or both). In what follows, $C(\cdot)$ stands for a choice function. If procedures do not matter, we get

$$C([A, i]) = C([A, j]) \neq \emptyset$$

for all $A \subset X$ and any $i, j \in \mathcal{N}, i \neq j$. This result could be called *consequentialist*, since, obviously, the only thing which is of relevance is the set of objects A (and not the aspect how A came about). *Non-consequentialism* would hold if for at least one $A \subset X$, one would have $C([A, i]) \neq C([A, j])$ for $i \neq j$, or for at least one $A \subset X$ and for at least one procedure $i \in \mathcal{N}$, $C([A, i]) = \emptyset$.

In the following sections, we wish to discuss non-consequentialist features of decision-making, procedural aspects in particular. We shall try to develop a theoretical framework that, we hope, will be able to deal with these characteristics appropriately. In section 2, we shall discuss another example, argue in favour of a non-consequentialist approach and introduce some basic structure. Section 3 presents definitions, axioms and offers a first result. Section 4 argues that information about how objects “fare” under different procedures should enter the process of choice. Section 5 introduces a finer structure where aspects of various production methods are considered as well. Section 6 proposes an ordering over shrinking procedures and section 7 concludes. All proofs are given in an Appendix at the end of the paper.

2 Another Example and the Basic Structure

We wish to start by discussing a situation that we have described before (Gaertner and Xu, 2004). It is a modification of an example given by Sen (1988). Let us consider a country with a finite set of newspapers, some highly political, others only marginally so. Among the first, there is one daily paper which is considered to be the government’s mouthpiece, but with undeniable editorial qualities nevertheless. The person we consider chooses this paper among all the available papers. Let us imagine that for some reason, the government decides to ban all papers except for its mouthpiece. The person we observe now decides to refrain from choosing. In other words, her choice set with respect to papers is the empty set. So if n is the mouthpiece and k stands for the procedural aspect of a government ban, we would have $C(\{\{n\}, k\}) = \emptyset$, $n \in S$, where S denotes the set of newspapers before government intervention.

Note that the newspaper example is just one case. A government may ration the supply of certain consumer goods under the pretext that resources are scarce or industry has only limited capacities. The reader will remember that under socialism there was a notorious scarcity of certain consumer goods. The state’s emphasis was on nurturing and extending heavy industry, military weapons in particular.

It seems that once such a procedural approach is taken, it is possible to follow various paths. One of the directions would be to argue that an alternative x , for which $C(\{x\}, i) = \emptyset$ for some $i \in \mathcal{N}$, should *never* be picked under the same i . In other words, for any $A \subset X$ such that $x \in A$, $x \notin C(A, i)$. One could say, however, that this requirement is too strict so that whenever there is at least one other alternative y available under procedure i for which it is not true that $C(\{y\}, i) = \emptyset$, then x would be choosable. This argument could be modified again by saying that an alternative x for which $x \notin C(\{x\}, i)$, would need a “sufficient or minimal number” of other objects around in order to be chosen. In our example above, one could reason that if a couple of other papers were left intact under the ban (i.e. under the given procedure i), mouthpiece n would perhaps still be picked. This last direction was followed at greater length in our earlier paper (Gaertner and Xu, 2004).

It also seems plausible to argue that the government mouthpiece would, perhaps, still be chosen by an individual if it was left as the only available newspaper after a major natural disaster or as the only paper available after a series of bankruptcies in the press industry due to financial mismanagement. This last observation points into one of the directions we are pursuing in this paper, namely that bits of information about how a certain alternative fares “choice-wise” under various procedures should be taken into consideration. The following sections will be more explicit.

Let us pause for a moment and discuss a fundamental objection to the approach on which we are about to embark. From the examples given above, it should be clear that some decision-makers (not all of them) seem to care about one or several characteristics of a choice procedure which indicates that these characteristics are of relevance for them. Shouldn’t these characteristics then be included in a proper description of an alternative or in a proper description of the consequences of an action? These consequences then should not only refer to the decision-maker alone but also to every other person that can be affected, both today and in future times.

This objection which reflects a consequentialist’s position raises the issue of a complete or all-encompassing description of an alternative or action. Could a more comprehensive description of alternatives and actions render the special emphasis on procedural aspects superfluous? An answer to this question involves several aspects. First of all, it may be difficult or impossible to have a full description of all relevant features of an alternative or all relevant aspects of the consequences of a decision-maker’s action. Informational constraints may be one reason for this, by no means the only one. There

are also practical reasons why alternatives may be incompletely described. But even if alternatives and actions could be fully specified, they may not be of ultimate interest. Some of the aspects that would render the description of an alternative or action complete depend on information that is so private that, in some cases, the decision-making agents themselves may not be fully aware of all the facets that affect their choices. Therefore, it may be difficult, perhaps impossible to determine which particular comprehensively described alternative or action has been chosen. What can be determined without much difficulty, however, is which *incompletely* described alternative or action has been picked. In other words, alternatives that are exchanged on markets, claims that are made in contracts and actions that are required in strategic situations are, because of informational reasons, necessarily incomplete descriptions. Baigent and Gaertner (1996, p. 241) write in a related context that “even a brief glance at the standard economics literature establishes beyond doubt that it is such partial descriptions (commodity bundles, tax schedules, etc.) that are the alternatives of ultimate interest”.

There is another aspect which has to be considered. Let us assume that the procedural characteristics have been integrated into a proper description of particular objects. Then at the time of choice, trade-off relationship among different characteristics would have to be considered. An analogy to the choice of a car may help where different features such as speed, luxury, gas consumption and other aspects are weighed against each other. However, it seems to us that in the case of procedural characteristics, such trade-off relationships very often are not considered. Children’s toys which were manufactured in prisoners’ camps appear to belong to a different category than “normal” toys. The same seems to hold for carpets woven by children or diamonds mined by children or women. Individuals may decide that these aspects are of no relevance for them. Consequently, there will be no consideration at all in terms of possible trade-offs between characteristics. On the other hand, these procedural aspects do matter for others, actually matter very much, and then again, there is no weighing among different features that describe particular objects or alternatives. In the latter case, it is possible to argue that procedural aspects are considered quite separately. They may even receive lexicographic priority. This then comes close to assigning them a separate role or status.

Having said all this, we have to ask how procedural aspects can be formalized adequately. Given sets X and \mathcal{N} as they were defined in section 1, let K be the set of all non-empty subsets of X and $K \times \mathcal{N}$ be the cartesian product of K and \mathcal{N} . The elements of $K \times \mathcal{N}$ will be denoted by $[A, i], [B, j]$, etc. The intended interpretation of $[A, i]$ is that the subset A is brought about or produced by the procedure i . A choice function C is a mapping from $K \times \mathcal{N}$ to $K \cup \{\emptyset\}$ such that for all $A \in K$, all $i \in \mathcal{N}$, $C([A, i]) \subseteq A$. In addition, we define $C([\emptyset, i]) = \emptyset$. Note that we shall allow $C([A, i]) = \emptyset$ for some $A \in K$. The emptiness of a choice set can be regarded as an

inaction. The individual considered refuses to choose anything from the set of choosable elements. It should be clear from our discussions so far that we interpret $C([A, i])$ as follows: given a subset A , which is brought about or produced by a procedure i , $C([A, i]) \subset A$ is the set of alternatives that the individual would like to pick from A given i . Let R be a weak ordering (reflexive, transitive and complete) over X . For all $A \in K$, define $\hat{C}(A, R) = \{x \in A \mid xRy \text{ for all } y \in A\}$. $\hat{C}(\cdot)$ specifies the set of best elements according to ordering R . Furthermore, we define $\hat{C}(\emptyset, R) = \emptyset$.

3 Definitions, Axioms and a First Result

As a first step, we concentrate on those elements x in A such that the choice set of the singleton $\{x\}$ under procedure i is empty. More precisely, given any procedure $i \in \mathcal{N}$, we collect all those objects $x \in X$ for which $C(\{x\}, i) = \emptyset$.

Definition 1. Given a choice function C , for all $i \in \mathcal{N}$, all $A \in K$, define $W(A, i) := \{x \in A \mid C(\{x\}, i) = \emptyset\}$.

$W(A, i)$ consists of all those elements x in A that under procedure i are not chosen from the singleton set. We now propose a way in which elements of $W(A, i)$ can be “treated” when the set of choosable elements A contains more than a single element. Analogous to previous discussions (Arrow (1959), Sen (1977)), the following notion of rationalizability of choice functions, which was first proposed in Gaertner and Xu (2004), is introduced in this extended framework.

Definition 2. A choice function C is *LP-rationalizable* iff for all $i \in \mathcal{N}$, there exists an ordering R_i over X such that for all $A \in K$, $C([A, i]) = \hat{C}(A - W(A, i), R_i)$ if $W(A, i) = A$ and $C([A, i]) = \hat{C}(A, R_i)$ if $W(A, i) \neq A$.

Definition 2 says that elements from $W(A, i)$ are potential candidates for choice as long as at least one other alternative outside of $W(\cdot)$ is available. This is a requirement that expresses a “limited protest”.

An example may help to clarify the implication of definition 2. Let $X \supset A = \{x, y, z\}$. Furthermore, let us assume that $W(A, i) = \{x, y\}$ according to definition 1. Then, according to definition 2, since $W(A, i) \neq A$, the issue of rationalizability concerns all three elements of A . If A only contained elements x and y , $C([A, i])$ would be equal to the empty set.

Our first axiom expresses the idea that if two options x and y are not picked from singleton sets, given procedure i , then if one of them is not chosen from set $A \subset X$, it continues not to be picked if the other option is added to set A .

Weak Protest Consistency (WPC): For all i in \mathcal{N} , all A in K , all x in A and all y

in X , if x and y are not chosen from singleton sets, given procedure i , then x is not chosen from $[A \cup \{y\}, i]$ whenever x is not chosen from $[A, i]$.

One may object to axiom WPC that it is already too strong. If alternatives x and y , let's say, are such that each of them is not picked from the corresponding singleton set, it would seem unnatural, so the argument goes, to choose nothing from the set $\{x, y\}$, when this set is generated by procedure i . We do not find this argument very convincing. If x and y , again, are newspapers and happen to be the government's mouthpieces, should one be willing to choose one of them just because of the fact that procedure i "was kind enough" to leave a choice between two papers?

The following axiom captures a similar intuition as contained in axiom WPC: if nothing is picked from a set A because of an objection against the underlying procedure, and if the option x is not chosen from the singleton set $\{x\}$, then x continues not to be picked if x is added to set A .

Protest–Expansion Consistency (PEC): For all i in \mathcal{N} , all A in K , all x in X , if $C([A, i]) = \emptyset$ and $C(\{x\}, i) = \emptyset$, then x is not chosen from $[A \cup \{x\}, i]$.

It is clear that axiom PEC is both conceptually and formally weaker than axiom WPC.

The next axiom is rather innocuous. It guarantees the non–emptiness of the choice set, given set A and procedure i , if there exists at least one element that does not belong to $W(A, i)$.

Non-empty Choice of No Protest Situations (NCNP): For all i in \mathcal{N} , all A in K , if there exist x, y in A such that not both objects belong to $W(A, i)$, then $C([A, i]) \neq \emptyset$.

The third axiom is a consistency requirement when the set of choosable options contracts. It is a straightforward modification (weakening) of Arrow's (1959) rationality condition.

Restricted Arrow Condition (RAC): For all i in \mathcal{N} , all A and A' in K , if $A' \subseteq A$ and there exists x in A' such that the choice from $\{x\}, i$ is not empty, then $C([A', i]) = A' \cap C([A, i])$ whenever $C([A, i]) \cap A' \neq \emptyset$.

In section 1, we asserted that traditional choice theory does not consider procedural aspects. This statement can be turned around by saying that conventional choice theory defines conditions of consistency independently of the underlying procedure. In Arrow's rationality condition from 1959, for example, it does not matter at all whether the transition from superset A to subset A' , let's say, was caused by procedure i or procedure

j or some other process, nor would the consistency requirement be affected in any way if the shrinkage from A to A' were generated by procedure i and the transition from A' to A'' were caused by some other procedure j . In other words, the traditional consistency conditions are stronger than our requirement since they are implicitly defined for all i, j in \mathcal{N} .

The first result we wish to offer is the following.

Theorem 1 (Gaertner and Xu (2004)). A choice function C is LP -rationalizable iff it satisfies axioms WPC, NCNP and RAC.

In this theorem, axiom WPC can be replaced by axiom PEC from above without affecting the result so that we have the following Theorem 1'. Its proof is given in the Appendix.

Theorem 1'. A choice function C is LP -rationalizable iff it satisfies axioms PEC, NCNP and RAC.

4 Information from Other Procedures

So far, we have confined our theoretical analysis to cases where only one procedure at a time is considered. However, as indicated in section 2, it may be important for a person to ask whether a certain alternative would, perhaps, not be chosen under several other procedures or even not be picked under any procedure at all. In our newspaper example, we argued that an individual may refuse to pick the only paper left after government has introduced a ban on all other political papers. Our individual may also decide not to buy the only political paper left if the other papers have been bought up by the government and turned into completely apolitical ones. Another case would be a situation where the rulers of a country declared that because of an emergency situation, it would be a waste of scarce resources if more than one daily paper (their own, of course) appeared. However, as mentioned before, the individual would, perhaps, continue to acquire the government's mouthpiece if the other papers had been destroyed by a major disaster or went out of business due to financial mismanagement. In other words, we now introduce some degree of interdependence among different procedures that permits additional information to enter the process of choice.

In line with the foregoing arguments, let us consider the following refinement. For all elements $x \in X$, we define the set of procedures $N(x)$ with the property that x is not picked from the singleton set. More formally, for all $x \in X$, $N(x) = \{i \in \mathcal{N} | C(\{x\}, i) = \emptyset\}$. Next, the individual is supposed to compare the cardinality of $N(x)$ with some x -specific threshold level $q(x) > 0$ which defines a level of tolerance for the choosing individual. In other words, if $\#N(x)$ is larger than $q(x)$, $x \in A$ has "failed sufficiently" under various procedures and will be deleted from further choice. For all

$A \in K$, $\Gamma'(A)$ collects all those elements from A that have this property. So we obtain $\Gamma'(A) = \{x \in A \mid \#N(x) > q(x)\}$. It is obvious that the introduction of the threshold level $q(x)$ only makes sense if the procedures that we consider can be clearly defined. Our various examples hopefully demonstrated that this is what we are having in mind. So in our newspaper example, the government ban on other papers, a government buy-out of other papers, financial mismanagement among the other papers leading to their bankruptcy are instances of clearly defined procedures.

Using the newly introduced notions above, we can now define the concept of LP' rationalizability that was introduced in Gaertner and Xu (2004).

Definition 4. A choice function C is LP' -rationalizable iff, $\forall i \in \mathcal{N}$, there exists an ordering R_i over X such that, $\forall A \in K$, $C([A, i]) = \hat{C}(A - W(A, i), R_i)$ if $W(A, i) = A$ and $C([A, i]) = \hat{C}(A - \Gamma'(A), R_i)$ if $W(A, i) \neq A$.

Furthermore we define

Protest Consistency Based on a Threshold (TPC): For all $i \in \mathcal{N}$, all $x \in X$ and all $A \in K$, if $x \notin C(\{\{x\}, i\})$ and $N(x)$ contains more elements than $q(x)$, then $x \notin C([A \cup \{x\}, i])$.

This condition says that an alternative that is rejected from the singleton set under a sufficient number of procedures will never be chosen when the set of choosable elements expands, under any procedure i .

The following result gives the characterization of a choice function being LP' -rationalizable.

Theorem 2 (Gaertner and Xu (2004)). A choice function C is LP' -rationalizable iff it satisfies TPC, WPC, NCNP and RAC.

We note that in Theorem 2, axiom WPC may be replaced by axiom PEC so that we have the following result. Its proof is given in the Appendix.

Theorem 2'. A choice function C is LP' -rationalizable iff it satisfies TPC, PEC, NCNP and RAC.

5 A Finer Structure

Up to now, we presented two aspects of non-standard rationalizability of a choice function originating from Gaertner and Xu (2004) by focusing on procedures or processes that shrink the universal set X of all conceivable alternatives to particular feasible subsets S, S', \dots of X . We did not discuss the aspect of production methods as such that give rise to alternatives contained in a feasible set and moreover, we did not look at

the interaction between those production methods and processes that shrink the set X to alternative feasible subsets of X . In order to do all this, we need a finer structure within which we hope to be able to carry out our analysis. Before embarking on this, we should be a little more explicit on what we mean by “the aspect of production methods as such”.

In standard microeconomic analysis, production of inputs and outputs is more or less entirely reduced to a technological process. This is justified in many cases but not in all situations since there are production methods that are not innocuous. Carpets woven by children, children’s toys manufactured in prisoners’ camps, gold mined by children and hard labour done by women are activities that cannot just simply be reduced to different forms of technology, since standards of human dignity and humanity and of safety are immediately involved. Quite a few people but, of course, not everybody has reservations against these forms of production. Coal mining in China, carried out by men, would be another case in point, though at the first glance, it looks rather harmless. These phenomena have to be put in relation to urgent wants, non-availability or scarcity of badly needed objects. Just reconsider our last example. Coal mining with high fatality rates may be viewed differently under severe shortages of energy supply than “under normal circumstances”. The fact that there is no general or unanimous view on these issues does not mean that they should be neglected. We now turn to these aspects and propose some analysis.

Let X again be a finite set of all conceivable alternatives. We now wish to describe these alternatives in more detail. Following Debreu (1959), we specify production methods that generate physical commodities or alternatives. Therefore, let T be the set of all relevant production methods. We assume that T can be partitioned into three non-empty subsets T^1 , T^2 and T^3 ($T = T^1 \cup T^2 \cup T^3$), with the interpretation that T^1 consists of those production methods that are *never* acceptable for a choosing individual, T^2 comprises those production methods that are *always* acceptable, and T^3 consists of those methods whose acceptability is contingent *on other things*. Consequently, a commodity or an alternative from the universal set X henceforth specifies the underlying production method in an explicit form. For example, we now distinguish between coal mined by men and coal mined by women (or children), carpets woven by women or by children, or road construction done by men or by women. For the ease of presentation, we assume that $X = O \times T$, where O stands for the set of all features of commodities or alternatives other than T . As a consequence, alternatives in X can be denoted by (x_0, τ) , (y_0, τ') , etc., where $x_0, y_0 \in O$ and $\tau, \tau' \in T$.

Next, we focus on possible procedures that give rise to a feasible subset from a given set of commodities or alternatives. For this purpose, let N stand for the set of procedures that narrow down a given set S to some proper subsets of S . We assume that N can be partitioned into procedures N^1 that we describe as interventionist procedures

(such as dictatorship, wars, and other kinds of imposed interferences) and N^2 that we decompose into interventionist but “not directly man-made” procedures (natural catastrophes, large-scale accidents) and non-interventionist procedures such as the market system or a given legal system. We assume that neither N^1 nor N^2 is empty and that they completely exhaust N , i.e., $N = N^1 \cup N^2$.

We now wish to consider a relationship between procedures that determine the availability of certain subsets and those generated subsets themselves. With K being the set of *all* non-empty subsets of X , we distinguish between elements from N^1 generating $K^1 \subset K$ and elements from N^2 generating $K^2 \subset K$, where normally $K^1 \cap K^2 \neq \emptyset$. In general, it may not be possible to generate all elements of K from either N^1 alone or from N^2 alone. Also, an interventionist procedure such as dictatorship may bring about subsets from K that would be highly unlikely under a market system or a legal system which belong to N^2 (an imposed rationing of commodities, for example). Given the distinction between N^1 and N^2 , it should be clear that all elements of the cartesian product $K^1 \times N^1$ and all elements of the cartesian product $K^2 \times N^2$ are potentially feasible.

Given our definition of X from above, we consider $A \subset O$ and $\tau \in T$. Then $(A \times \{\tau\})$, $(A' \times \{\tau'\})$ are subsets of X , where $\tau, \tau' \in T$ are the underlying production methods. These subsets can be generated by shrinking procedures from either N^1 or N^2 . So we may, for example, have triples of the form $(A \times \{\tau\}; i)$ with $i \in N^1$ and $(A' \times \{\tau'\}; j)$ with $j \in N^2$, where subset $(A \times \{\tau\})$ belongs to K^1 and $(A' \times \{\tau'\})$ belongs to K^2 . A choice function C chooses nothing or picks an alternative $(a, \tau) \in (A \times \{\tau\})$ from $(A \times \{\tau\}; i)$. For example, we may find that coal mined by women is an acceptable choice in the case of a war (an interventionist procedure from N^1) but unacceptable under elements from N^2 .

More generally, given $O \times T$, K^h and N^h , $h \in \{1, 2\}$, a choice function C is a mapping from $K^h \times N^h$ to $K^h \cup \emptyset$ such that for all $(A \times \{\tau\}) \in K^h$, all $i \in N^h$, $C(A \times \{\tau\}; i) \subseteq (A \times \{\tau\})$. In addition, we define $C(\emptyset, i) = \emptyset$. In the present framework, we again allow $C(A \times \{\tau\}; i) = \emptyset$ for some $(A \times \{\tau\}) \in K^h$, $h \in \{1, 2\}$. Let R be a weak ordering (reflexive, transitive, and complete) over X . For all $(A \times \{\tau\}) \in K^h$, we define $\hat{C}(A \times \{\tau\}, R) = \{(a, \tau) \in (A \times \{\tau\}) | (a, \tau) R (b, \tau) \forall (b, \tau) \in (A \times \{\tau\})\}$. $\hat{C}(\cdot)$ specifies again the set of best elements, given ordering R . Furthermore, we define $\hat{C}(\emptyset, R) = \emptyset$.

In the sequel, we wish to propose another notion of rationalizability of choice functions, reflecting our newly extended framework. We start by introducing an axiom that can be considered as rather uncontroversial. However, it would not lead us very far within our current project.

- A.1. For all $(A \times \{\tau\}; i)$ from either $K^1 \times N^1$ or $K^2 \times N^2$, if $\tau \in T^1$ and $(a, \tau) \in (A \times \{\tau\})$, then $(a, \tau) \notin C(A \times \{\tau\}; i)$.

Once a commodity or option has been produced by a production method from T^1 , it will never be chosen, independent of whether the feasible set of objects was generated by an element from N^1 or N^2 .

We shall now elaborate the idea that an individual may find the variety of choosable objects, left over by a shrinking procedure, unsatisfactory or “too thin”. The individual has set up a cardinality requirement or threshold level below which objects that do not meet this requirement are not acceptable for choice. Let $k(a, \tau; i)$ be such a threshold level. It is object-specific and will, in general, depend both on $\tau \in T$ and on $i \in N^h$. For example, the threshold level may, *ceteris paribus*, be more demanding for $\tau \in T^3$ than for $\tau \in T^2$ and may also, *ceteris paribus*, be higher for $i \in N^1$ than for $i \in N^2$.

In the following, we shall restrict our analysis to elements $(A \times \{\tau\}; i) \in K^1 \times N^1$ with $\tau \in T^2 \cup T^3$. A similar analysis can, of course, be done for elements from $K^2 \times N^2$. In order to avoid complications, we shall henceforth assume that (1) $(O \times \{\tau\}; i) \in K^1 \times N^1$ for all $\tau \in T^2 \cup T^3$ and all $i \in N^1$, and (2) for all $A \subseteq O$, all $\tau \in T^2 \cup T^3$ and all $i \in N^1$, $(A \times \{\tau\}; i) \in K^1 \times N^1$.

A.2. Restricted Protest Consistency (RPC): $\forall \tau \in T^2 \cup T^3, \forall (A \times \{\tau\}; i) \in K^1 \times N^1$, if $(a, \tau) \in (A \times \{\tau\})$ and $\#(A \times \{\tau\}) < k(a, \tau; i)$, then $(a, \tau) \notin C([A \times \{\tau\}; i])$.

An explanation for this type of choice behaviour was given above. If an element from $(A \times \{\tau\})$ does not meet its alternative-specific threshold level, it will not be acceptable for choice.

We next define what we shall call “ k -cardinal rationalizability”.

Definition: A choice function C is k -cardinally rationalizable iff for all $\tau \in T \setminus T^1$, there exists an ordering R_i over K^1 such that for all $(A \times \{\tau\}; i) \in K^1 \times N^1$, $C([A \times \{\tau\}; i]) = \hat{C}(A \times \{\tau\} - \Omega(A, \tau; i), R_i)$, where $\Omega(A, \tau; i) = A \times \{\tau\}$ if $\#(A \times \{\tau\}) < k(a, \tau; i)$ and $\Omega(A, \tau; i) = \emptyset$ if $\#(A \times \{\tau\}) \geq k(a, \tau; i)$.

Elements that do not meet the cardinality requirement are eliminated from $(A \times \{\tau\})$ and are, therefore, no longer objects of potential choice. They comprise the set $\Omega(A, \tau; i)$.

Next, we reformulate Sen’s (1977) well-known expansion consistency condition β for the present purposes.

A.3. Expansion Consistency (EXC): For all $\tau \in T \setminus T^1$, for all $(A \times \{\tau\}; i), (B \times \{\tau\}; i) \in K^1 \times N^1$ with $(B \times \{\tau\}) \subseteq (A \times \{\tau\})$, if $(a, \tau), (b, \tau) \in C([B \times \{\tau\}; i])$, then $(a, \tau) \in C([A \times \{\tau\}; i])$ iff $(b, \tau) \in C([A \times \{\tau\}; i])$.

If elements (a, τ) and (b, τ) were chosen from set $B \times \{\tau\}$, given $i \in N^1$, then either both elements are to be chosen from the larger set $A \times \{\tau\}$ or neither of the two will be picked.

Furthermore, we consider

- A.4. Restricted Contraction under $N^1(\alpha^{N^1})$: For all $\tau \in T \setminus T^1$, for all $(A \times \{\tau\}; i)$, $(B \times \{\tau\}; i) \in K^1 \times N^1$, if $(a, \tau) \in (B \times \{\tau\}) \subseteq (A \times \{\tau\})$ and $(a, \tau) \in C([A \times \{\tau\}; i])$, then $[\#(B \times \{\tau\}) \geq k(a, \tau; i) \longrightarrow (a, \tau) \in C([B \times \{\tau\}; i])$.
- A.5. Consistency of Empty Choice (CEC): For all $\tau \in T \setminus T^1, \forall (A \times \{\tau\}; i)$, $(B \times \{\tau\}; i) \in K^1 \times N^1$, if $(B \times \{\tau\}) \subseteq (A \times \{\tau\})$ and $C([A \times \{\tau\}; i]) = \emptyset$, then $C([B \times \{\tau\}; i]) = \emptyset$.
- A.6. Non-Emptiness (NE): For all $\tau \in T \setminus T^1$ and for all $(A \times \{\tau\}) \subseteq (O \times \{\tau\})$, if there exists $(a, \tau) \in A \times \{\tau\}$ such that $k(a, \tau; i) \leq \#(A \times \{\tau\})$, then $C([A \times \{\tau\}; i]) \neq \emptyset$.

A.4 formulates contraction consistency within the new structure. If (a, τ) was picked from the set $A \times \{\tau\}$ under procedure $i \in N^1$, (a, τ) will still be chosen from subset $B \times \{\tau\}$ if the cardinality requirement $k(a, \tau; i)$ is met.

A.5 requires consistency of no choice when one goes from larger sets to smaller sets, but note that this is required only for procedures i within N^1 .

A.6 says something very obvious: if for any set $A \times \{\tau\}$, at least one element (a, τ) meets the threshold level, the choice set from this set is nonempty.

We can now formulate the following result; its proof is given again in the Appendix.

Theorem 3. A choice function C is k -cardinally rationalizable iff it satisfies RPC, EXC, α^{N^1} , CEC and NE.

6 An Ordering over Different Procedures

Do people have orderings over procedures? At the first glance, it does not appear plausible that individuals have a ranking over procedures per se. People judge procedural aspects on the basis of their implications for choice. This paper discussed several aspects, the shrinkage of sets in particular which involves a restriction of choice, limiting a person's freedom to choose. A priori, it is not clear at all, at least for us, how an ordering over procedures could be derived. Our final point in this paper is to make at least one small proposal.

In the second half of the previous section, we focussed on the set N^1 of interventionist man-made procedures, related to production methods from $T^2 \cup T^3$. We introduced a threshold level for richness or variety of choice $k(a, \tau; i)$. We argued that elements from the set $(A \times \{\tau\})$, given $i \in N^1$ and $\tau \in T^2 \cup T^3$, should only be picked if "sufficiently many" choosable objects were around. If $k(a, \tau; i)$ is an indirect measure and requirement for richness, it should vary with the type of procedure. One could, for example, argue that the stronger the outside intervention with respect to the availability of objects, the stricter the variety requirement should be, or put differently, the stronger

the interference, the higher an individual will set the threshold level $k(a, \tau; i)$. So this level should be lower for elements from N^2 , the set of procedures which do not come about through direct man-made intervention. Thus, if $(a, \tau) \in (A \times \{\tau\})$ meets the cardinality requirement under elements i from N^1 , it meets the cardinality requirement under elements j from N^2 a fortiori. These bits of information may allow us to construct an ordering over procedures. For a given $\tau \in T^2 \cup T^3$ and $(a, \tau) \in (A \times \{\tau\})$, we wish to define a preference relation \mathcal{R} over procedures $i \in N^1$ and $j \in N^2$ in the following way: $\langle j \rangle \mathcal{R} \langle i \rangle$ iff $k(a, \tau; j) \leq k(a, \tau; i)$. Due to its construction, the relation \mathcal{R} will be complete and transitive.

One can introduce finer partitions, both with respect to N^1 and N^2 , and use the threshold level again as the basis for an ordering over shrinking procedures. Thus, if we have $N^1 = N^{11} \cup N^{12}$ and $N^2 = N^{21} \cup N^{22}$ with indices i, j, l, m referring to subsets $N^{11}, N^{12}, N^{21}, N^{22}$ respectively, one can establish a complete ordering on the basis that in general $k(a, \tau; i) > k(a, \tau; j) > k(a, \tau; l) > k(a, \tau; m)$. Thus, the strictest level requirement is attached to elements from N^{11} (strong interventionist measures taken by a dictator or oligarchy, let's say) and the weakest level requirement is attached to elements from N^{22} (shrinkages due to the market mechanism or to instances of democratic legislation). It is unlikely that there will be a level requirement at all in the latter case. Since the levels $k(a, \tau; i)$ are object-specific and consider the underlying production method, they do not represent abstract index numbers. Of course, more complicated constructions seem possible.

7 Concluding Remarks

In various choice situations, an individual may have good reasons to consider non-consequentialist features of decision problems, procedural aspects in particular. This non-consequentialist attitude towards decision making is certainly different from the standard approach in the theory of rational choice and calls for an unconventional investigation into an individual's choice behaviour. In traditional microeconomic analysis, an "all-encompassing" utility function takes all possible consequences into consideration, for everyone who can be affected, including future generations. This is quite demanding. The present paper has proposed a different framework that enables us to examine various issues of non-consequentialism reflected in an individual's choices.

In our theoretical framework, we have analyzed several types of choice behaviour that cannot be rationalized within the standard approach. A common thread in this non-standard approach is the notion that an individual may choose to pick nothing from feasible sets of alternatives if those feasible sets are brought about in particular ways that are deemed unacceptable. This emptiness of a choice set is in sharp contrast to the non-emptiness of a choice set, almost universally assumed in the standard framework.

Our approach reflects the procedural concern of the individual under consideration: if the way that gives rise to various feasible subsets is not acceptable according to the individual's subscribed view, the individual may register a protest by refusing to choose any alternative from the given feasible subset even though some alternatives in the set would have a positive value for this person. Starting from this "position", we have moved onward to other considerations which require more information. Richness of available choice is one of them, information on how an object fares under different procedural conditions is another. Depending on the context, we have proposed various notions of rationalizability of a choice function that we have then characterized axiomatically.

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Appendix

Proof of Theorem 1'. It can be checked that if a choice function C is LP -rationalizable, then it satisfies axioms PEC, NCNP and RAC. Therefore, we have only to show that if a choice function C satisfies PEC, NCNP and RAC, then it is LP -rationalizable.

Let C be the choice function that satisfies PEC, NCNP and RAC. Let $i \in \mathcal{N}$. First, we note that for all $A \in K$, if $C(\{a\}, i) = \emptyset$ for all $a \in A$, then, by PEC, $C([A], i) = \emptyset$. Note also that the above observation is independent of any underlying binary relations. If $C(\{x\}, i) = \emptyset$ for all $x \in X$, then define the binary relation R_i over X as follows: xI_iy for all $x, y \in X$. Clearly, given PEC, the R_i thus defined is LP -rationalizable. Now, suppose that for some $x \in X$, $C(\{x\}, i) \neq \emptyset$. Given NCNP, $C([X], i) \neq \emptyset$. We first define X_1, X_2, \dots, X_k as follows: $X_1 = C([X], i)$, $X_2 = C([X - X_1], i)$, \dots , $X_k = C([X - \cup_{j=1}^{k-1} X_j], i)$ and k is such that $W(X_k, i) \subseteq X_k$, $C([X_k], i) \neq \emptyset$ and $C([X - \cup_{j=1}^k X_j], i) = \emptyset$. Now, define the binary relation R_i as follows: for all $x, y \in X$, xR_iy iff $x \in X_m$ and $y \in X_n$ where $m \leq n$. Clearly, R_i as defined above is reflexive, transitive and complete. Next, we show that C is LP -rationalizable; that is, for all $A \in K$, $C([A], i) = \hat{C}(A - W(A, i), R_i)$ if $W(A, i) = A$ and $C([A], i) = \hat{C}(A, R_i)$ if $W(A, i) \neq A$.

Consider $A \in K$ such that $W(A, i) = A$. Let $A = \{a_1, \dots, a_p\}$. Since $W(A, i) = A$, for all $a \in A$, $C(\{a\}, i) = \emptyset$. In particular, $C(\{a_1\}, i) = \emptyset$. Then, by the repeated use of PEC, we must have $C([A], i) = \emptyset$. On the other hand, $\hat{C}(A - W(A, i), R_i) = \hat{C}(\emptyset, R_i) = \emptyset$. Therefore, in this case, $C([A], i) = \hat{C}(A - W(A, i), R_i)$.

Now, consider $A \in K$ such that $W(A, i) \neq A$. Clearly, there exists $x \in A$ such that $C(\{x\}, i) = \{x\}$. It is then clear that $W(X, i) \neq X$. From the definition of R_i , clearly, $C([X], i) = \hat{C}(X, R_i)$. It is also clear that, from the definition of R_i , $C([X_j], i) = \hat{C}(X_j, R_i)$ for all $j = 1, \dots, k$. We now show that $\hat{C}(A, R_i) = C([A], i)$ in this case.

(1) Let $a \in C([A], i)$. By RAC, noting that $C(\{x\}, i) = \{x\}$, it must be true that $a \in C(\{a, x, y\}, i)$ for all $y \in A$. Suppose to the contrary that $a \notin \hat{C}(A, R_i)$. Then, from the definition of R_i , there exists $b \in A$ such that $b \in X_p$ and $a \in X_q$ with $p < q$. Note that $C([X_p], i) = \hat{C}(X_p, R_i)$. We must have $b \in C([X_p], i)$. Consider $C(\{a, b, x\}, i)$. If xP_ib , then $x \in C([X_m], i)$ where $m < p$. By RAC, $\{x\} = C(\{x, a, b\}, i)$, a contradiction. If xI_ib , then $x \in C([X_p], i)$. By RAC, $\{x, b\} = C(\{x, a, b\}, i)$, a contradiction. If bP_ix , then, by RAC, $\{b\} = C(\{x, a, b\}, i)$, another contradiction. Hence, $a \in \hat{C}(A, R_i)$.

(2) Let $a \in \hat{C}(A, R_i)$. Then, from the definition of R_i , for all $y \in A$, aR_iy . Let X_p be such that $a \in C([X_p], i)$. Then, noting that $C(\{x\}, i) = \{x\}$, by RAC, we must have $a \in C([A], i)$. Therefore, C is LP -rationalizable. ■

Proof of Theorem 2'. The necessity part of the theorem can be easily checked. We show the sufficiency.

Let C be the choice function that satisfies TPC, PEC, NCNP, and RAC. Let $i \in \mathcal{N}$.

We distinguish two cases: case (i) $C([X, i]) = \emptyset$ and case (ii) $C([X, i]) \neq \emptyset$. In case (i), we define the binary relation R_i over X as follows: $xI_i y$ for all $x, y \in X$. Then, by PEC, the R_i defined above is LP' -rationalizable. In case (ii), let $X' = X - \Gamma'(X)$. By TPC and NCNP, $C([X', i]) \neq \emptyset$. Define X'_1, X'_2, \dots, X'_k as follows: $X'_1 = C([X', i])$, $X'_2 = C([X' - X'_1, i])$, \dots , $X'_k = C([X' - \cup_{j=1}^{k-1} X'_j, i])$ and k is such that $W(X'_k, i) \subseteq X'_k$, $C([X'_k, i]) \neq \emptyset$ and $C([X - \cup_{j=1}^k X'_j, i]) = \emptyset$. The remainder of the proof of the result is similar to that of Theorem 1'. Therefore, we omit it. ■

Proof of Theorem 3. It can easily be checked that if a choice function C is k -cardinally rationalizable, then it satisfies axioms RPC, EXC, α^{N^1} , CEC and NE. Therefore, we only have to show that if a choice function C satisfies RPC, EXC, α^{N^1} , CEC and NE, then it is k -cardinally rationalizable.

Note that at the beginning of our formal analysis, we had defined $C(\emptyset; i) = \emptyset$ and $\hat{C}(\emptyset, R) = \emptyset$, where R is a weak ordering.

Let C be the choice function that satisfies RPC, EXC, α^{N^1} and CEC. Let $(A \times \{\tau\}; i) \in K^1 \times N^1$ with $\tau \in T \setminus T^1$. We distinguish the following two cases: case (i) $C(O \times \{\tau\}; i) = \emptyset$ and case (ii) $C(O \times \{\tau\}; i) \neq \emptyset$. In case (i), due to CEC, $C(B \times \{\tau\}; i) = \emptyset$ for all $(B \times \{\tau\}) \subseteq (O \times \{\tau\})$. In other words, no $(a, \tau) \in (B \times \{\tau\}) \subseteq (O \times \{\tau\})$ meets its cardinality requirement $k(a, \tau; i)$. We define the binary relation R_i over $O \times \{\tau\}$ as follows: $(a, \tau)I_i(b, \tau)$ for all $(a, \tau), (b, \tau) \in O \times \{\tau\}$. Clearly, R_i is reflexive, complete and transitive. It is also clear from the definition of $\Omega(A, \tau; i)$ that $\Omega(A, \tau; i) = A \times \{\tau\}$ for all $A \times \{\tau\} \subseteq O \times \{\tau\}$ so that $\hat{C}(A \times \{\tau\} - \Omega(A, \tau; i), R_i) = \emptyset = C(A \times \{\tau\}; i)$ for all $(A \times \{\tau\}) \in K^1$. Hence C is k -cardinally rationalizable.

In case (ii), we define A_1, A_2, \dots, A_p as follows: $A_1 = C(O \times \{\tau\}; i)$, $A_2 = C(O \times \{\tau\} - A_1; i)$, \dots , $A_{p-1} = C(O \times \{\tau\} - \cup_{m=1}^{p-2} A_m; i)$, $A_p = O \times \{\tau\} - \cup_{m=1}^{p-1} A_m$, where p is such that $A_{p-1} \neq \emptyset$, $O \times \{\tau\} - \cup_{m=1}^{p-1} A_m \neq \emptyset$ and $C(O \times \{\tau\} - \cup_{m=1}^{p-1} A_m; i) = O \times \{\tau\} - \cup_{m=1}^{p-1} A_m$ or $C(O \times \{\tau\} - \cup_{m=1}^{p-1} A_m; i) = \emptyset$. Now define the binary relation R_i over $O \times \{\tau\}$ as follows: $(a, \tau)R_i(b, \tau)$ iff $(a, \tau) \in A_m$ and $(b, \tau) \in A_n$, where $m \leq n$. From its construction, R_i is reflexive, complete and transitive. Also, from the construction and by NE, we have $C(O \times \{\tau\}; i) = \hat{C}(O \times \{\tau\} - \Omega(O, \tau; i), R_i)$. Next we show that $C(A \times \{\tau\}; i) = \hat{C}(A \times \{\tau\} - \Omega(A, \tau; i), R_i)$ for all $(A \times \{\tau\}; i) \in K^1 \times N^1$. Let $(A \times \{\tau\}; i) \in K^1 \times N^1$. Let $(a, \tau) \in C(A \times \{\tau\}; i)$. Given that $(a, \tau) \in C(A \times \{\tau\}; i)$, obviously, $\#(A \times \{\tau\}) \geq k(a, \tau; i)$. Hence, $(a, \tau) \notin \Omega(A, \tau; i)$. By EXC and α^{N^1} , we are back in the classical case of contraction and expansion consistency. From the definition of R_i , $(a, \tau)R_i(b, \tau)$ for all $(b, \tau) \in A \times \{\tau\}$. Therefore, $(a, \tau) \in \hat{C}(A \times \{\tau\} - \Omega(A, \tau; i), R_i)$. This shows that $C(A \times \{\tau\}; i) \subseteq \hat{C}(A \times \{\tau\} - \Omega(A, \tau; i), R_i)$. Suppose next that $(a, \tau) \in \hat{C}(A \times \{\tau\} - \Omega(A, \tau; i), R_i)$. From the definition of $\Omega(A, \tau; i)$ and by condition RPC, for all $(c, \tau) \in \Omega(A, \tau; i)$, $(c, \tau) \notin C(A \times \{\tau\}; i)$. From the definition of R_i , there exist m, n with $m \leq n$ such that $(a, \tau) \in A_m$ and for all $(b, \tau) \in$

$(A \times \{\tau\} - \Omega(A, \tau; i)), (b, \tau) \in A_n$. Then, by α^{N^1} and EXC, $(a, \tau) \in C(A \times \{\tau\}; i)$ follows from standard rationalizability. Hence, $\hat{C}(A \times \{\tau\} - \Omega(A, \tau; i), R_i) \subseteq C(A \times \{\tau\}; i)$. Therefore, $C(A \times \{\tau\}; i) = \hat{C}(A \times \{\tau\} - \Omega(A, \tau; i), R_i)$. That is, C is k -cardinally rationalizable. ■