Public Economic Expenditure Theory

III Summer School in Public Economics

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John Conley
Vanderbilt University
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1. Introduction

- What is public economics?

Public economics is the study of the interaction between the government and the economy.

- Musgrave gives four functional roles for government.

A. Provision of Goods and services.
   e.g. Education, roads, defense, parks,
B. Redistribution
   e.g. Welfare, social security, food stamps
C. Stabilization
   e.g. Low inflation, low interest rates, low unemployment, steady growth, stable markets
D. Regulation
   e.g. Clean air and water, restraining monopoly, liability, and contract enforcement

To obtain resources to fill these roles, the government relies on four main sources:

A. Taxes
   e.g. Income tax, sales tax, VAT, property taxes, tariffs, excise taxes
B. Debt
   e.g. Savings bonds, revenue bonds
C. Confiscation
   e.g. Draft, nationalization
D. Gifts
   e.g. Donations (land, art), foreign aid, time of volunteers.

This division does not really reflect the research agenda as it is being pursued today.

Regulation is more a branch of IO except where it touches on issues of externalities
Stabilization is more a branch of macro and many issues in redistribution fit better into macro or social choice.

- A better division now would between **expenditure** and **taxation**. Unfortunately, there is also a division between **empirical** and **theoretical** public economics, especially on the expenditure side.

On the theory/expenditure side, research focus on structural causes of and potential solutions to market failure. The following organization is based of the one proposed by Marcus Berliant:

- Externalities
- Public Goods
- Local Public Goods
- Public Choice/Voting/Political Economy
- Cost Benefit Analysis

The empirical/expenditure side tends to be organized around various classes of expenditure. Particularly active areas include:

- Poverty/Welfare
- Health
- Education
- Social Security
- Environmental

The division between theory and empirical on the taxation side is not as stark. There are a number of different mythological approaches including partial equilibrium, GE, CGE, simulations, and OLG and other dynamics. Major topics include

- Optimal Commodity taxation
- Optimal income tax
- Fiscal competition
- Tax evasion
- Effect of taxes on Savings/SS

This is not an exhaustive list.

The plan of this talk is to go very briefly over some the basic structure of expenditure theory, and then discuss some of the more active research areas.

2. Externalities

We will focus on theory of externalities. Our organization will be as follows:

1. Definition
2. Pigouvian approaches
3. Coasian approaches
4. Strict liability
5. Regulation and mandates
6. Arrovian approaches
7. Other topics: Stock externalities, Transboundry pollution

1. What is an externality?

Definition: An *externality* the effect of any action other than buy or selling commodities that affects the welfare of any other agent.

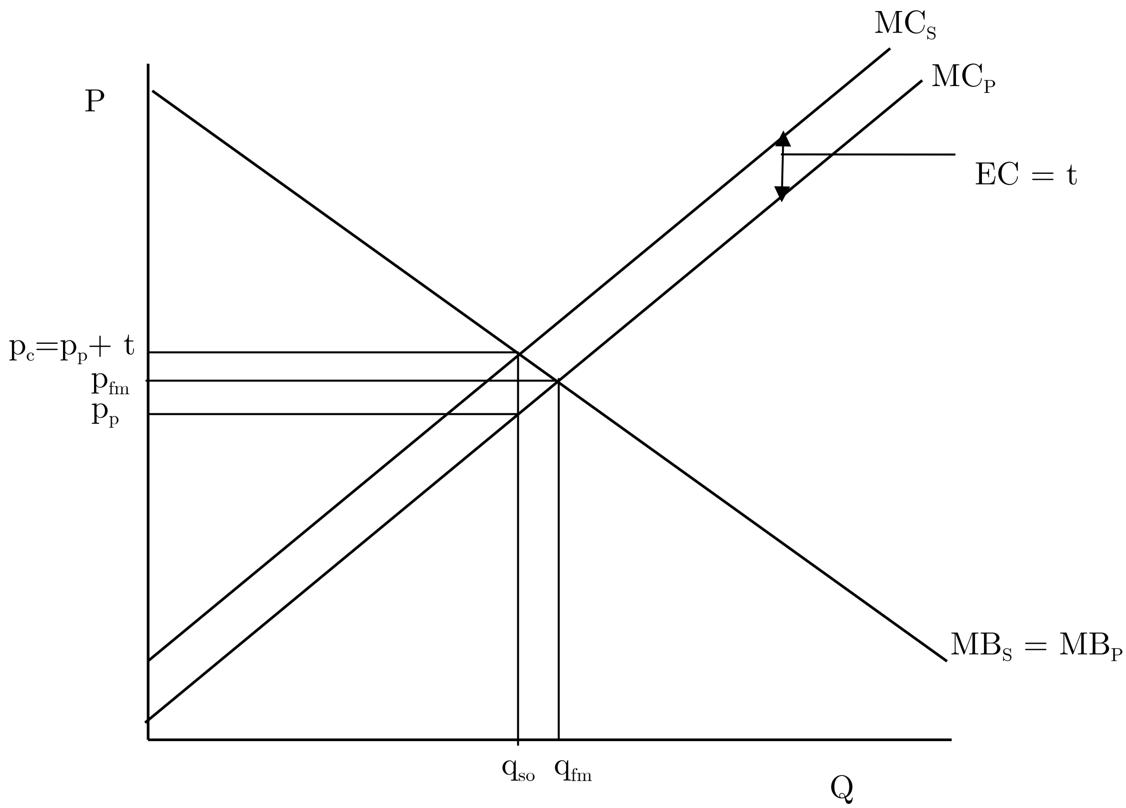
The effects can be positive or negative, and come as a result of either consumption or production choices.

e.g. water pollution, smoking a cigarette, painting your house, getting immunized.

Pecuniary externalities can occur when market actions affect the welfare of other agents. For example, if I bid up the price of a painting at an auction, I am certainly affecting this welfare of the other bidders, but only through equilibrium prices. Thus, this is not a true externality since it does not result in inefficiency, merely a reallocation of wealth.

2. The Pigouvian approach.

This is the undergraduate approach to externalities. The basic argument is that the presence of externalities drives a wedge between private and social costs and benefits. The solution is the impose taxes or subsidies to cause the agents to artificially internalize these social costs. Graphically:



<u>Other examples: positive externality in production, and positive and negative externalities in consumption.</u>

The point is that efficiency requires that social costs and benefits be the same. The externality-producing agent does not take into account the costs imposed on other agents.

Real world examples of Pigouvian taxes:

- Cigarette taxes
- Gasoline taxes
- Educational subsidies
- Research subsidies
- Agricultural subsidies?

Limitations of Pigouvian taxes:

- Estimating externalities
- Fixed tax over all levels of production and consumption and over time (counter examples

- Tax is on output and not externalities themselves. This means that avoidance technologies are ignored.
- Like all price-based solutions, it depends on convexity. We shall see that this is questionable.

3. Coasian approach

Coase argues the following. Markets fail in the presence of externalities because **markets are incomplete.** In turn, markets are incomplete because **property rights** are not fully assigned.

In the canonical case of air pollution, air is a valuable commodity and that is a vital input into production processes. It also has competing uses in consumption, in particular, we may wish to breath the air.

The problem is that since no one owns the air, the factory simply **appropriates** air until its marginal value in production is **zero**. This is hardly surprising.

Suppose no one owned **labor**. Then factories would go around gather slaves until the marginal value of the next slave was equal to cost of catching him. The point is that any commodity with no property rights is used past the point where social costs equal social benefits.

The solution is quite obviously to simply **assign property rights** to air and other similar commodities and let the market allocate them efficiently.

- Should we assign property rights away from the agent who causes the externalities?

It turns out that this is not a meaningful question. Consider the following examples:

- I walk into an elevator with a cigar. Have I caused an externality?
- I build my house under the main runway at O'Hare Field. Have I caused an externality?

The answer is **no** in both cases. **Externalities are reciprocal.** It takes at least two parties, the actor and the victim to produce externalities. Cause and blame is an ethical concept, not an economic idea. Often it relates to what you think is a **fair status quo**.

- This all leads us to the "Coase theorem".

A word of caution: There is no Coase theorem *per se*, certainly not in any of his papers. To the extent there is one, there is no proof in any event. This is a very hotly debated issue amongst public economist.

I think of the Coase Theorem as being like **love.**

- We can't agree on exactly what it is.
- Some even deny its existence.
- Others devote all their thought and energy to it, even if they don't understand it.
- It is elusive, yet it can't be ignored.
- To even attempt a definition is perhaps to cheapen it.

With this caveat, let me give you my understanding of the Coase Theorem.

Weak Coase Theorem: If transactions costs are zero, then any assignment of property rights leads to a Pareto efficient outcome.

Illustration: Consider a Laundry and a Steel Mill.

- Show that no matter how property rights are assigned, PO is achieved.
- Show the same with avoidance technology (Consider a scrubber.)
- Show how not assigning property right is not the same. (Suppose that you bribed a factory to shut down. You still have not bought clean air as another factory can set up.)

- An implication is that we simply choose to assign property rights in a way that suits our sense of ethics and fairness. The final outcome will be first best regardless.

- The requirement that transactions costs be zero is standard and not particular to markets with externalities. It may perhaps be stronger here, especially if there are many players. The necessity of collective action may generate systematically high transaction costs.

- Suppose that transactions costs were positive. Then the best course is to assign the rights away from the least cost avoider. Courts are often asked to make these determinations.

Now consider a little old lady and Donald Trump.

- Different assignments of property rights seem to make a difference. Why?

- Income effects. Note that never the less, all outcomes are efficient.

Strong Coase Theorem: If transactions costs are zero, and incomes effects are zero, then any assignment of property rights leads to the same Pareto efficient outcome.

- Some have argued that the Coase Theorem is either a tautology or false.

If Coase is saying that when property rights are transactions through a Pareto optimal mechanism, the outcome is Pareto optimal, then clearly his is correct. It is not clear what contribution is in this case.

If Coase is saying that if TC=0 and property rights are fully assigned, then any mechanism to transact property rights will be Pareto optimal, then he is clearly wrong.

The problem is Coase is silent on this point from a formal standpoint, and his examples and discussions suggest a number of things.

The two that are most prominent are some sort of "market" for externalities and simple bilateral or multilateral exchanges that are not mediated via a market.

4. Strict Liability

Definition: *Strict liability* is a system enforced by courts that require that any agent who imposes a negative externally on any other pay full compensation.

This is a kind of offshoot of the Coasian approach. The idea is that every agent is responsible for any and all damage they impose. The thought is that this causes agents to internalize any externality they impose.

- Question is it important that agents are compensated for the damage they experience? ----NO! In fact, compensation (without a transfer of property rights) can lead to its own inefficiency.) The important thing is that the polluter pays the cost of damage. It would work just as well to pay this to general fund.

In an ideal world this has some appeal. If a firm had to pay for any damage it generated, it clearly would take this into account as a cost. It might even get firms to internalize hidden long-term damage like dumping toxic waste into the water table.

In practice this falls on many difficulties.

- Transactions costs are high, so only major damage will ever end up in court, and even that in a second best way
- Damage is hard to estimate, and if it can't be accurately for seen, firms will not internalize it.
- Firms may calculate a chance of bankruptcy in the future that causes them to inefficiently discount future liabilities.
- It gives pollutee's no incentive to avoid damage. (Why not put a house at the end of a runway?)
- The property rights it establishes are only partial. It gives agents the right not to be damaged, but the polluter can never fully acquire the right to pollute.
- Does not effectively treat positive externalities.

5. Standards

Here the government decides by fiat that an agent must undertake specific actions to mitigate pollution.

Examples:

- All cars have catalytic converters
- Banning freeon or DDT
- Regulating the content of effluent from paper mills
- Auto emissions or mileage standards
- Requiring that people drive on the right side of the road
- Requiring childhood immunizations.

The advantages of this approach are that it is cheap and easy to implement from the government's standpoint, and also does not depend on markets or convexity.

The disadvantages are that

- Standards are a broad and inefficient brush
- No incentive to take advantage of avoidance technologies or improving trades once the standard is meet
- It is difficult to find the right standard (measurement)

6. Arrovian approaches

The Arrovian approach picks up Coase's basic observation that markets are incomplete.

Arrow then takes a more formal general equilibrium approach to completing markets that exposes several important details.

At a less formal level Arrow assumes:

- Zero transactions costs
- Price taking behavior

Beyond this, he makes two fundamental assumptions:

(C) Convexity of preferences and production sets (Also nonsatiation, bounded above production, and complete and transitive preferences, as is typically required in GE models)

(M) Universality of markets

He claims the following results:

TH1: M → CE is PO (1WT)

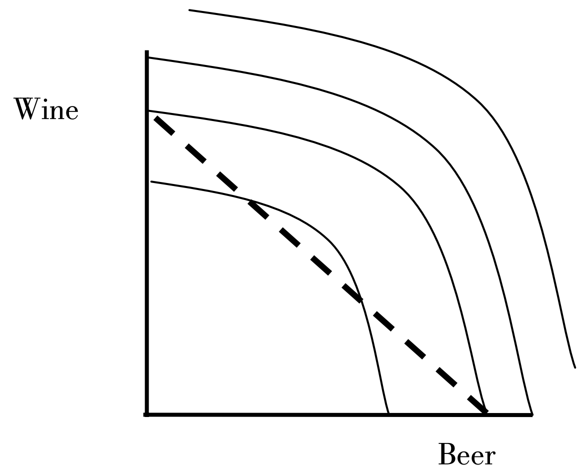
TH2: M+C → Any PO is a CE for some initial endowments (2WT)

TH3: C → There exists a CE (not C is not necessary, just sufficient.)

Alternatively,

(C') There are no individuals who are large relative to the economy.

Intuitively, this mitigates the effects of any Nonconvexities. For example, suppose individual preferences were nonconvex between Beer and Wine. At the prices given by the dashed line, the agent would want only beer or only wine. If production technology is convex however, these prices would support production of some wine and some beer. Clearly, this is not equilibrium. Suppose there were a thousand agents, then at these prices, half could have only wine, and half only beer (or any other proportion, for that matter.) Thus, as the number of agents gets large, by altering the proportion on each of the extreme consumption points, we can get arbitrary close to any point on the convex production set supported by these prices. Similar arguments hold for nonconvex production.



Note, however, this does not allow for unbounded IRS on the part of firms. In this case, the equilibrium would be for only one firm to get large relative to the economy and thus the conditions of C' would fail.

TH4: If C is replaced by C', Theorems 2 and 3 hold approximately.

TH5: M → CE is in the core (if CE exists)

TH6: M+C' → As the economy gets large, every core allocation is approximately a CE

Two points about TH6

- M in this case implies that the number of markets is fixed and in particular does not grow as the economy grows.
- Is transactions cost are positive, the core can remain large (why?)

Arrow markets

Since the issue is clearly that M is not satisfied when there are externalities, Arrow proposes to complete the markets and then simply apply well-known GE results like those above.

Consider an ordinary production economy with N firms and K commodities. We use the following standard notation:

y_{jk} the amount of good k produced by firm j.

p_k the price of good k.

$Fj(y_1,\ldots,y_K)=0$ defines the production possibilities for firm j.

Now suppose that the production activities produced externalities. To be concrete, suppose that good k was coal, which is used as an input by firm j to produce steel. Using coal generates smoke that damages the production of laundry by firm i.

Arrow's clever trick is to define a set of artificial commodities as follows:

y_{ijk} firm $i's$ *observation* of the net production by firm j of commodity k.

p_{ijk} Price paid by firm i per unit of *observation* of the net production by firm j of commodity k.

$F_i(y_{111},\ldots,y_{ijk},\ldots,y_{NNK})=0$ defines the production possibilities for firm i.

- Thus the each firm's production of every commodity is an argument in every other firm's production possibilities.
- If there are no externalities, the effect of these arguments is zero.
- If there is a negative externality the effect is negative. For example. If firm j burns coal, then keeping the production equation in balance requires either less clean laundry be is produced by firm i or the inputs be increased. Given the signing convention, Formally this implies:

$$\frac{dF_i}{dy_{ijk}}>0$$

- If the use of an input generated a positive externality the sign would be reversed.

One very important factor to note is that all the observations have to agree. In other words, burning a unit of coal generates one unit of observational commodity for all the firms in the economy. Formally:

$$y_{ijk}=y_{\hat{i}jk}\forall i,\hat{i},j,k$$

To put this another way, **observations are a strictly jointly produced commodity**.

Since joint production does not affect the validity of standard GE results, what Arrow has accomplished is the following:

Take an economy with arbitrary positive or negative externalities. Simply transform the economy by changing the K dimensional commodity space to an N^2K in a completely mechanical way and extent the price space and production functions as outlined. No more needs to be done to apply standard theorems.
Now consider prices:

$p_{11k} = 20 \leftarrow$ Firm 1's observation of its own consumption of good k is interpreted as the direct price. In this case, the ordinary price of coal in the market is 20 per unit.

$p_{21k} = 10 \leftarrow$ Firm 2 is a laundry and each unit of coal firm 1 burn imposes a cost of 10.

$p_{21k} = -5 \leftarrow$ Firm 3 is a farm which benefits from the extra rain that all the smoke particles in the air generate.

$p_{41k} = 0 \leftarrow$ Firm 4 is a restaurant that neither benefits nor is damaged by smoke.

$\sum p_{i1k} = 25 \leftarrow$ This is the net price that firm one must pay to buy a unit of coal.

Notice that the observational prices above are very much like Pigouvian taxes. From the standpoint of firm one, it pays taxes or receives subsidies for over unit of input or output in accordance with the externalities they generate for other agents in the economy.

It is also the case that firms that experience externalities are compensated through the price system for their effects.

Given convexity, these prices are fully decentralizing. Suppose we are at a CE for this Arrow economy and that firm 1 consumes 300 units of coal in equilibrium. Then the following happens. Firm 1 knows that to consume these units of coal, it jointly produces 300 units of observational commodities for each of the other firms that must find a buyer (no free disposal). Thus, the effective prices of coal per unit given these prices is 25.

As a price taker firm 2 considers the price of 10 per unit of is happy to take the -300 units of coal observations (recall coal is an input and so is negative by convention) and accept the net income of 3000. Moreover, given convexity, if this is an equilibrium price, then 300 is where the marginal cost of an additional unit of coal consumption by the steal mill to the laundry is exactly 10. Thus, 300 units is the profit maximizing choice.

Again, the point is that given convexity and price taking behavior, these Arrow prices are fully decentralizing.

One can easily imagine extending this to include consumers. Now the commodity space would be "every consumers or producers observations of the consumption or production of every commodity by every other consumer or producer". Formally, there is not change.

Apparently, Arrow markets work, and this provides some theoretical foundation for Coase.

Four observations:

- Recall that in the case of TH6, above, M required not only that markets are universal, but fixed for all sizes of the economy. Clearly this is not the case with Arrow markets. The number of markets increases with the square of agents
- To work, Arrow markets require price-taking behavior. However, by construction, each commodity only has one meaningful buyer and seller. Thus, by construction, these markets are thin. Price taking behavior is a very strong requirement.
- Arrow markets do not encourage the use of abatement technologies. If the steel mill is a price taker, he by definition does not believe that if he were to put on a scrubber, his observational prices would change. He therefore has not incentive to do so.
- Is convexity a valid assumption? Not so clear. See work of Starrett, Boumal, Boyd, Conley and Smith

Direct externality markets

The question becomes, can we set up alternative markets that do not fall victim to fundamental Nonconvexities?

Following an informal suggestion in Laffont's book, Boyd, Conley and Smith (in different combinations attempt to define direct externally markets.)

By direct externally markets we mean we must specifically define such things as smoke, sewage, noise, CO_2 as externalities (really, public bads) and set up dedicated markets to them. This is more difficult that Arrow's approach at the front end as it requires that we think about the real economics at hand, not simple perform a mathematical transformation of the economy.

What we show is that problem of externalities essentially reduces to a problem of public goods allocation. Thus, convexity is not longer an inconsistent assumption, and all the usual theorems of GE work.

7. Stock externalities and transboundary pollution

So far we have treated only a static world. Many externalities are only interesting in a dynamic context. I will say only a few words about this.

Stock Externalities

For example, if it is indeed the case that global build up of CO_2 will cause global warming and ultimately serious environmental consequences, it is not the current emissions that are really the concern. It is to sum of emissions across all time. That is to say, it is the stock, not the flow of externalities that causes the damage.

What are the implications of this?

Suppose that it is very costly to substantially reduce the flow of externality quickly. Then to the to the extent that the flow is very small compared to the stock, you should be willing to let any necessary adjustment phase be longer. This is especially true if the damage is uncertain. However if the flow is great compared to the stock, or you think that you are approaching a stock level with discontinuous negative effects, this does not hold.

Transboundary pollution.

There is not much work in this area, but it is very interesting. Essentially, the idea is that pollution is a local externality rather than a pure externality, but that the externality spillover across borders. This means that the locally that hosts a polluter gets to export part of the damage.

This opens many interesting questions about regulatory competition and the potential desirably of policy harmonization.

By the same token, it makes us realize that if that if externalities are localized, it may very well be that different types of agents have different willingnesses to tolerate pollution. It would not be optimal to force a uniform standard over everybody.

There are many open research questions here.

Public Goods

1. What is a public good?

Definition: A *public good* is a positive externality that is intentionally produced and enters into the objectives or constraints of all other agents in identical quantities.

At a more operational level, we can distinguish classes of goods according to two properties.

Non-Rivalry: The entire quantity of a good that is produced is available to all agents simultaneously. In particular, one agent's consumption of a good does not diminish anther agent's ability to consume the same good.

For example, if the local television station broadcasts an hour's worth of programming, everyone can turn on their TV's and watch that program. The fact that I watch the program does not in any way diminish your ability to watch the program.

In contrast, a hamburger is a **purely rival good**. If I eat it you can't. This is the polar opposite case of a purely non-rival good.

Of course there are also intermediate cases like swimming pools. This is a **partially rival good.** If I use the pool, it is not prevent you from also using the pool, but I crowd you to some extent and so I diminished the value of the good to you.

We will consider partial rival goods when we discuss local public goods.

Non-Excludability: It is not possible to prevent any agent from simply appropriating any of the good that happens to cross his path. Put another way, once produced, there are no enforceable property rights to the good.

For example: One cannot detect who is listening to a given radio broadcast, and so one cannot exclude people from doing so. Similarly, anyone who walks by can listen to an open-air concert. It may also not be feasible to prevent people from picking the berries that grow in large state parks.

In contrast, if I want to eat a hamburger, I must buy it from Ronald McDonald. If I want to get HBO, I must pay my Cable provider. If I want to here N'sync at the superdome, I must buy a ticket.

- Traditionally, a public good has been considered to have both these properties.
- This confuses issues. Consider the following:
    - NR-NE: Network TV broadcasts
    - NR-E: Cable TV broadcasts
    - R-NE: Berries in a public park

- R-E: Berries in a supermarket

What we see is that Rivalry is an intrinsic quality of the good. Excludability has more to do with the specifics of institutions and is by no means fundamental to the good.

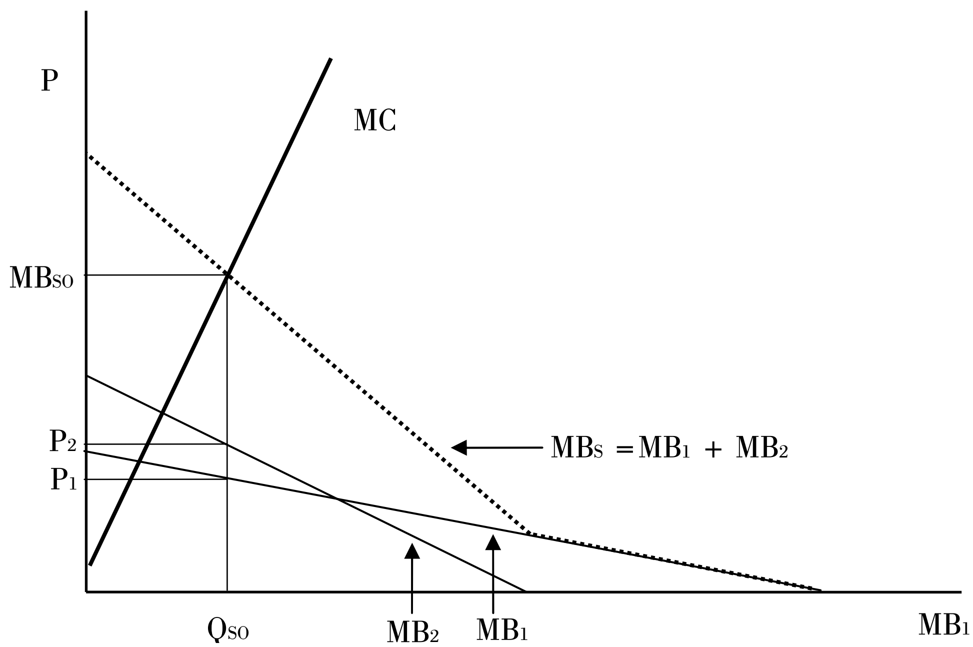
Thus, from a formal standpoint, it is more sensible to separate these two consideration.

We will therefore take the position that a public good simply one that is non-rival in consumption.

2. Partial equilibrium Lindahl equilibrium

What is the social marginal benefit of producing a banana? It depends of who consumes it, of course. If the banana is efficiently allocated, however, it goes to the person with the highest marginal value, and so the social MB equals the MB of this consumer.

What is the marginal benefit of producing an hour of radio broadcast? It depends on who listens. Because this is a non-rival good, many people can do so simultaneous. Therefore the total benefit to society equals the sum of the benefits over each of the listeners. We can see this in the following graph:



In this picture, we have two agents, 1 and 2. Each has a marginal benefit curve. To find the social MB of radio broadcasts, we add the marginal benefits of each of the agents together

- Note that this means the for public goods, the aggregate demand curve is found by summing up the individual demands *vertically*. In contrast, for private goods, the aggregate demand is found by summing up the individual demands *horizontally*
- The intersection of the MC and the sum of the MB's defines the Lindahl equilibrium quality, and is socially optimal.
- Each agent is asked to pay his MB for the social optimal quantity. This is called his Lindahl price.
- Note that each agent has a personalized price, but consumes the same quantity of public good. In contrast, with private goods, each agent pays the same price, but consumes a personalized quantity. In this sense, public and private goods are dual.

- Lindahl prices are sometimes called Lindahl taxes. Viewed this way, these taxes are consistent with the *benefit theory of taxation* in contrast to the *ability to pay theory of taxation.* Share of the cost of the public good each agent pays is proportional to the marginal benefit he received.
- If we assume agents are price takers, Lindahl prices are decentralizing. For each agent, maximizing his own welfare requires that he demand public good up to the point that this personalized price equals his own MB. By construction, this takes place at the social optimum. For the firm producing pubic good, maximizing profit require producing pubic good up to the point that his producer price (equal to the sum of the agents' Lindahl prices) equals MC. By construction, this also takes place of the social optimum.

3. Kolm Triangle

Now let us begin to consider this from a general equilibrium standpoint. In private goods exchange economies, we can represent this in an Edgeworth box. In public goods economies, the analogous representation is called the *Kolm Triangle.*

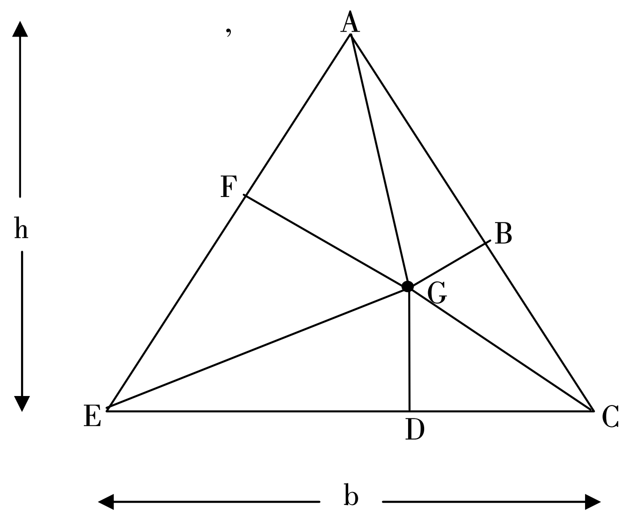
We consider an economy with:

- two agents, 1 and 2
- one private good, x
- one pubic good, y
- unit linear production technology for public good.

Agents are endowed with private good: Ω_1, Ω_2 and so the material balance constraint is:

$$\Omega_1 + \Omega_2 = y + x_1 + x_2$$

Before proceeding we need to prove an important theorem about equilateral triangles. Consider such a triangle with height, h, and base, b, and then choose an arbitrary point, G, within or on the boundary of the triangle. Then draw a set of perpendicular lines to each of the sides and another set of lines to each of the vertices.



Theorem: *The sum of the length of the perpendicular to any point G in the equilateral triangle equals the height, h.*

<u>Proof:</u>

We must show FG + DG + BG = h. First note the following:

$$\triangle ACE = \triangle AGE + \triangle CGE + \triangle AGC.$$

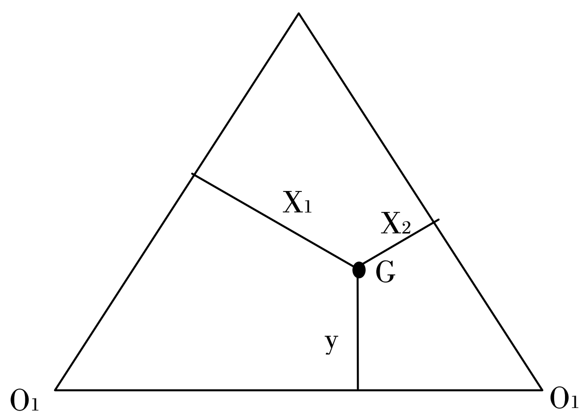
But given the formula for the area of a triangle is $\frac{1}{2}b \times h$, we can write this:

$$\frac{1}{2}b \times h = \frac{1}{2}b \times FG + \frac{1}{2}b \times DG + \frac{1}{2}b \times BG.$$

Divide through by $\frac{1}{2}b$, and we have the result.

□

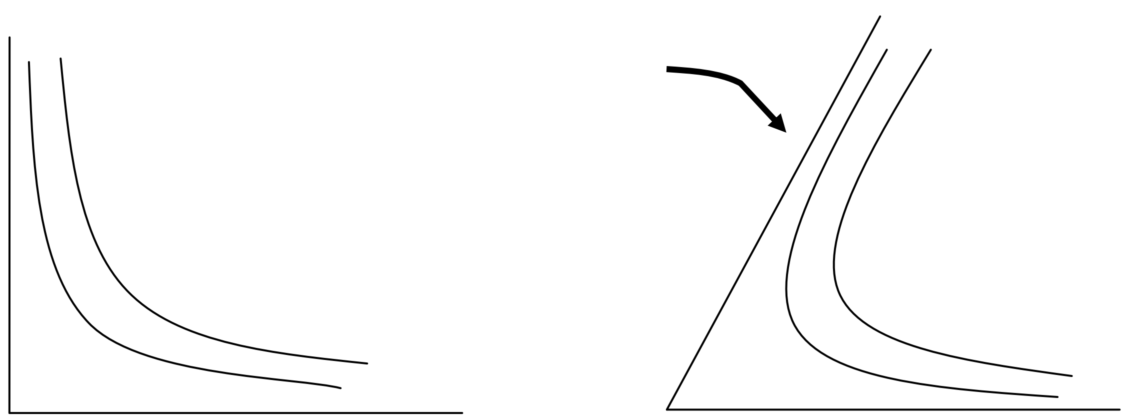
Now we turn the way that feasible allocations are represented in the Kolm triangle:



We now read the left vertice as being like the "origin" for agent 1 and the right for agent two.

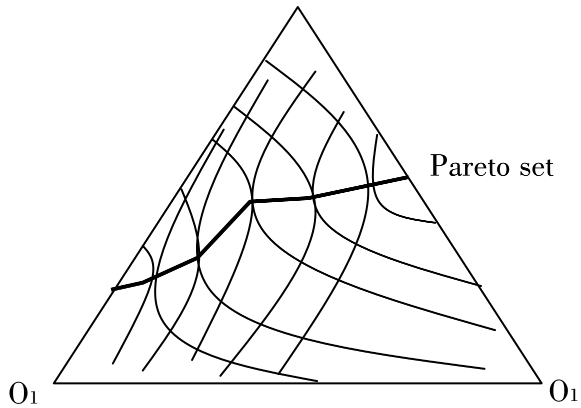
- The length of the left perpendicular is taken as the private goods consumption of agent 1, x_1
- The length of the right perpendicular is taken as the private goods consumption of agent 2, x_2.
- The length of the bottom perpendicular is taken as the joint public goods consumption.
- The key observation here is if we draw the size of the triangle such its height equals the sum of the endowments, then every point G is a feasible allocation since then the following equation is satisfied:

$$\Omega_1 + \Omega_2 = h = \text{(the sum of the perpendiculars)} = y + x_1 + x_2$$

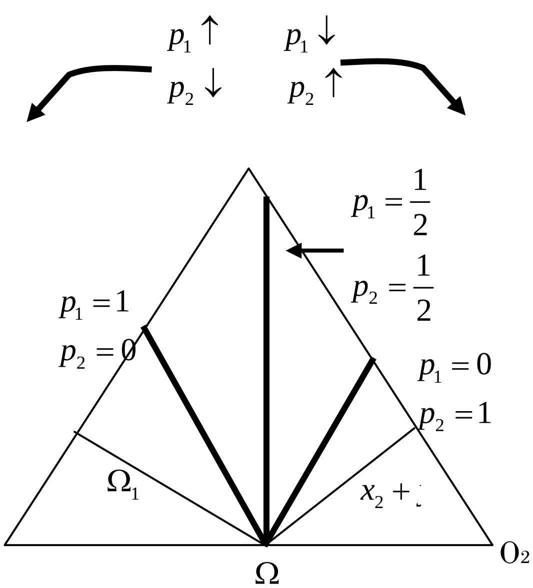


Preferences are simply squeezed from the by 30 degrees, but otherwise look just the same in the Kolm triangle.

- What do perfect complements look like?
- What do perfect substitutes look like?
- What does monotonicity require?



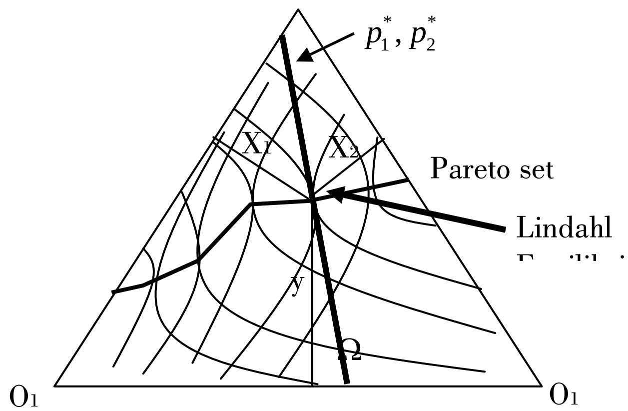
The contact curve is the set of tangencies between the two agent's indifference curves, just as in the Edgeworth box.



Now consider how we represent prices.

- A budget line through the endowment point that is parallel to the right vertice means that agent 2 pays one unit of private good per unit of public good. You can see this since the budget line generates a smaller equilateral triangle with the origin for agent 2 and there for any point along this budget line has the property that $x_2 + y = \Omega_2$. By a also observe that along this right most budget line, the all the allocations have the property that $x_1 = \Omega_1$. Thus, we see that the implied prices are: $p_1 = 0, p_2 = 1$.

- By parallel argument, the leftmost budget line has implied prices of $p_1 = 1, p_2 = 0$.
- Not surprisingly, the budget line exactly between these two (the budget line perpendicular to base) has implied prices of $p_1 = \frac{1}{2}, p_2 = \frac{1}{2}$.



This picture depicts a Lindahl equilibrium. The slope of the budget line represents the relative prices that, by construction, add to 1, the marginal cost of the pubic good. At the LE prices, both agents maximize utility at the same level of public good, but consume different levels of private good in general.

- Note that LE need not be unique.
- It is easy to show **1WT** and **2**WT using the Kolm triangle.
- What would be envy free and efficient allocations in this picture?

4. Samuelson and other General equilibrium approaches

Samuelson's major contribution was to show how to include pubic goods in a model of a general equilibrium economy. The most important thing that falls out of this are the so-called Samuelson conditions. Since Samuelson, many other economists have proposed variations of this approach and proven many additional results.

Lindahl equilibrium and Samuelson Conditions

Samuelson's model:

Private goods:	x_j	$j = 1, \ldots, n$
Public goods:	x_j	$j = n+1, \ldots, n+m$
Agents:	$U^i(x_1^i, \ldots, x_{m+n}^i)$	$i = 1, \ldots, s$
Production:	$F(x_1, \ldots, x_{m+n}) = 0$	

Social Planner's problem:

$$\max_{x_1^1, \ldots, x_j^i, \ldots, x_{m+n}^s} W(U^1, \ldots, U^s) \quad s.t.$$

$x_j = \sum_{i=1}^{s} x_j^i$	Private goods ($j = 1, \ldots, n$)
$x_j = x_j^i$	Public goods($j = n+1, \ldots, n+m$)
$F(x_1, \ldots, x_{m+n}) = 0$	Production

Notice:

- Private goods are purely rival. Thus, the sum of consumption over all the agents must equal exactly the total production of each private good.
- Public goods are purely non-rival. Thus each agent consumes exactly the total production of each public good.
- This approach requires the statement of a welfare function that weighs the well-being of each agent against each other. It turns out however, that for the major result, this interpersonal utility comparison is not necessary.

After working with the FOC's you get the followings:

1. $\frac{U_j^i}{U_r^i} = \frac{F_j}{F_r} \qquad i = 1, \ldots, s \quad r = 1, \ldots, n$ Private good efficiency

2. $\sum_{i=1}^{s}\frac{U_{n+j}^{i}}{U_{r}^{i}}=\frac{F_{n+j}}{F_{r}}$ $\quad j=1,\ldots,m \quad r=1,\ldots,n$ Public good efficiency

3. $W_{i}U_{k}^{i}=W_{q}U_{k}^{q}$ $\quad i,q=1,\ldots,s \quad k=1,\ldots,n$ Social Optimality

- The first is the standard MRS=MRT condition for private goods
- The second is the Samuelson condition. It says that the sum of the MRS's equals the MRT for public goods. This is just what we saw in the graphical representation. Note that neither of these two conditions depends of the welfare function.
- The third says that the incremental effect of each unit of private good on each agent's utility times his incremental social welfare weight must equal across agents. This tells how private good should be distributed. Note that there is not similar condition for pubic good since there is no distributional issue with pubic goods.
- This tells us nothing about prices or decentralization.

Cost share, Ratio, and other equilibrium notions

Local Public Goods

1. Overview

The fundamental problem with public goods provision is that we get free riding. This is both because

- Exclusion is sometimes quite hard to implement.
- Agents have an interest in hiding their willingness to pay for public goods.

Tiebout noticed, however, that many public goods are not provided by the central government, but by states and localities instead.

Examples?

Schools
Sewers
Police
Fire
Parks
Universities
Roads

What is different about this?

Agents may not have any real choice about what country they live in, but they can choose their state and city. Consider the choice facing agents over where to live:

Each location offers:

- A bundle of services such as schools, fire protection, roads, parks, and so on of a certain quality. There may also be other amenities offered by a location such as good restaurants, weather, city architecture, proximity to mountains or oceans, political, religious or cultural climate.
- A set of tax obligations including property, sales, and income taxes.

In effect, this is a market. There are public goods bundles offered by localities, and agents must agree to pay the prevailing tax price to gain access.

Tiebout argued that agents would look across the menu of these offers and "vote with their feet" in choosing the best offer.

Notice that in this context, exclusion is possible, and revelation of willingness to pay takes place when an agent chooses a place to live.

This revelation is just like what takes place the private goods market. (Consider a pizza as an example.)

2. Tiebout's idea:

Tiebout explored the problem of *local public goods*. Sometimes these are also called

Impure public goods
Congestible goods
Semirival goods
Club goods

In general these are goods that are somewhere between the extremes of pure public (non rival) goods and pure private (rival) goods.

What makes a public good impure?

- Crowding or congestion. It might be that the more people that share a public good, the less benefit they all get. However, this benefit decreases less quickly that the population increases (as it would for private goods).
  - Swimming pools
  - Schools
  - Roads
  - Parks
  - Disneyland
  - Concert (capacity constraint, a special case)

- Benefit decreases with distance. It might be that proximity to the public good makes a difference.
  - Fire stations
  - Hospitals
  - Police
  - Parks?

He argues that if

1. Agents are fully and costlessly mobile
2. Agents have full information
3. There are many communities
4. No spillovers
5. Employment is not a factor in location
6. For each pattern of tastes there is an efficiently sized community
7. City managers maximize profits

A. We get efficient provision of LPG's

B. In addition, agents will sort in to jurisdictions by taste.

Let's talk about each of the assumptions:

(1) This is just like saying no transactions costs. If it is violated, as it surely is in real life for this and every other market, we get less efficiency. However, we get much closer to the first best.

(2) If agents are uninformed, then prices will not equilibrate, and we will not get full efficiency. This is also true of other markets.

(3) "Many" means that the markets are thick. No community has a monopoly that it can use to exploit agents of a given taste. Otherwise, prices will be higher than is competitive. Is this true? How many Paris's or New York's are there? Cities may also offer unique bundles of natural and cultural amenities (a harbor, a mountain, a language group, as in Athens). We would expect competitive forces to operate less effectively here and for such cities to be less efficient. Cities with lots of substitutes must be efficient or they will loose their residents.

(4) Do cities have spillovers? Sure. Crime? Roads, Suburbanite enjoying city amentias? Pollution? Tiebout's model can't deal these. These lead to market failures just as externalities do in general. How important are these

(5) It turns out that further investigations have shown this not to be needed.

(6) This is key. I will talk about this more formally below. This is really the definition of LPG's it says that the optimal jurisdiction size for whatever reason is less than the entire population. Thus, efficient provision of public good requires that agents break up into smaller jurisdictions that then compete with each other for residents. Otherwise we are back in the pure public goods problem.

(7) City managers, or more generally, group organizers, must compete with each other for this to work. They play the role of "firms" here. It might be that they seek to maximize population, or the welfare of their residents, or the profit (what they can charge in taxes less what it costs to provide the bundle of public goods) (by the way, this is like corruption). In all cases, taxes and LPG provision are the tools they have to work with. It turns out not to matter much which of these objections they seek to maximize.

The results say two things.

(A) We get efficiency. It turns out to be more complicated than this. We have to work about many details. We will discuss these more below. But the general insight is true. We do find the following:

- Localities that raise taxes lower property values
- Localities that raise service or have high quality public goods have high property values.
- Communities with more competitors tend to be more efficient at providing service.

(B) There is some tendency to have agents sort by taste. People with kids who care about schools, go to suburbs, Single people live in cities. The elderly often move to warm locations. People are reluctant to move away from cultural communities. However, in other cases they mix. We will see why this might be true.

3. Formal first approach:

Consider following model:

One private good x (more are easily added)
One public good y (more are easily added)

Agents:

i= 1, . . .I

<u>Anonymous Crowding:</u>

Each agent is one some type

t=1,...,T

There are N_t agents of type t in the economy. Thus, the population can be described as

$(N_1, ..., N_t, ..., N_T) = N$

Agents form jurisdictions that make up a *partition* of the population. That is, each agent is in one and only one jurisdiction. A given jurisdiction is described as

n^j = the number of agents in jurisdiction j.

The number of agents in each jurisdictions j is given by the list:

$n = (n^1, ..., n^j, ..., n^J)$

Agents care about three things: public goods in the jurisdiction, private goods and the number of agents in the jurisdiction:

$U_t(x, y^j, n^j)$.

Production:

F(y, n)

gives cost in terms of private good of producing y public good with n agents in jurisdictions. Thus, crowding can also take place in production.

Examples:

- Swimming pools
- Standing in line
- People in your class?
- People at a concert
- Cars on the road.

There are two leading candidates for a notion of equilibrium. One is called Nash equilibrium and incorporates the idea that jurisdictions cannot prevent agents from joining. The other is called the Core and incorporates the idea that clubs form and can choose their own members.
Thus, suppose that all agents are identical and have the following utility function:

$$U(x, y, n). = x + y^{\frac{1}{2}} - \frac{n^2}{c}$$
$$F(y,n) = y$$

Lets work out what the social optimum.

$$MAX \ \omega - \frac{y}{n} + y^{\frac{1}{2}} - \frac{n^2}{c}$$

FOC:

$$\frac{-1}{n} + \frac{y^{\frac{-1}{2}}}{2} = 0$$
$$\frac{y}{n^2} - \frac{2n}{c} = 0$$

Solving:

$$\frac{1}{n} = \frac{y^{\frac{-1}{2}}}{2} \rightarrow \frac{n}{2} = y^{\frac{1}{2}} \rightarrow y^*(n) = \frac{n^2}{4}$$
$$\frac{y}{n^2} = \frac{2n}{c} \rightarrow \frac{c}{2} y = n^3 \rightarrow n^*(y) = (\frac{c}{2} y)^{\frac{1}{3}}$$

Solving we get:

$$y^* = \frac{c^2}{256}$$

$$n^* = \frac{c}{8}$$

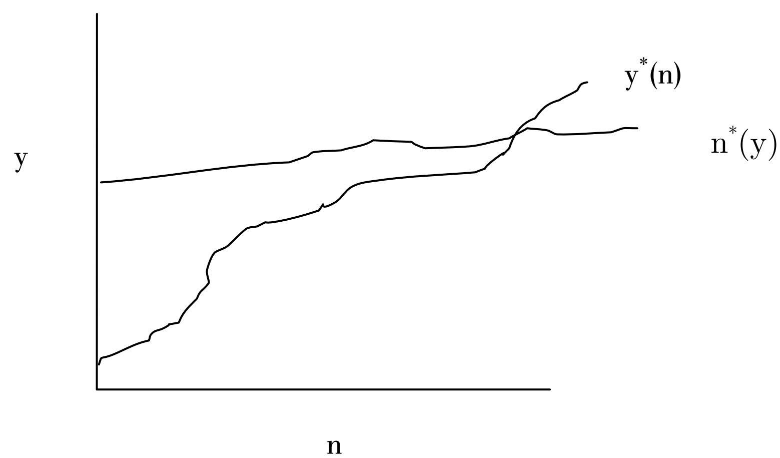
e.g. if c=64, n*=8 and y*=16

So we have optimal group size and optimal public good provision.

Here we can see two factors at work:

- For a given level of public good, the more agents you let in, the more widely you can share the costs.
- For a given level of public goods, the more agents you let in the more crowded you are.

These balance.



How does this become and equilibrium outcome? We have to worry about how what sort of equilibrium we are thinking about. There are three leading candidates: Nash, Core and Tiebout. Nash equilibrium and incorporates the idea that jurisdictions cannot prevent agents from joining, the Core and incorporates the idea that clubs form and can choose their own members, and Tiebout is more of a market notion in which prices decentralize actions

Let's think about Nash equilibrium first: Nash a set of strategies for each agent is equilibrium is there is no benefit for any agent to unilateral deviation. More completely: Suppose that agent x assumes that all the other agents have chosen a strategy and will keep this fixed. Then the best choice for agent x is also to keep his current strategy.

To investigate this, we have to worry about what the strategies are of the agents and how this translates to payoffs. Suppose the following: Let $(n^1, \ldots, n^J, y^1, \ldots, y^J)$ be an initial state for the economy.

Suppose that the only strategic choice of any given agent was which jurisdiction to choose. Also suppose that when an agent joins a jurisdiction, and he pays an equal share of public goods costs.

Examples:

- Cities
- Public dances
- Health clubs
- Open membership cultural and social clubs
- Churches?

Consider the example above and substitute in the constraint:

$$U(x, y, n) = \omega - \frac{y}{n} + y^{\frac{1}{2}} - \frac{n^2}{c}$$

Given this sharing rule we get the following

$$\frac{dU}{dy} = \frac{-1}{n} + \frac{y^{\frac{-1}{2}}}{2}$$

$$\frac{dU}{dn} = \frac{y}{n^2} - \frac{2n}{c}$$

Letting c=64, we get:

$$y^* = \frac{64^2}{256} = 16$$

$$n^* = \frac{64}{8} = 8$$

Is the socially optimal configuration Nash equilibrium?
Are there others?
What if preferences are such that large cities give the most per capita utility, small towns the second most, and medium places the third?
What if there is not the right number of agents to divide into optimally sized jurisdictions?
What if you expanded the strategy set? How?
What if you allowed for coalitional deviations.
What would happen to the social optimum if you added more types of agents? Homogeneity? Optimal club type for each agent?

Now let's think about the Core. Here, we allow exclusion of agents and also allow for more intelligent coalitional action. We say an allocation (public goods, private goods, and partition of agents) is in the core if there does not exist a "blocking coalition" to the allocation. A blocking coalition is a group that can break away for the grand coalition, and, using only their own resources, make every member of the blocking coalition better off.

The motivation is that no individual or group will agree to be taken advantage of when the outside opportunities are more attractive.

Examples:

- Exclusive clubs
- Universities
- Partnerships
- Firms
- Marriages
- Groups of friends

Let's get some practice with the core in a simple example. We consider a game in characteristic function form. This is a reduced form of the more elaborate economy I gave above in which we focus on the value of coalitions.

Formally, V(n) is taken as the total wealth available to a coalition of size n that it can distribute across its members.

For example, suppose:

$V(1)= 1$
$V(2)= 3$
$V(n)= 2n$ for $n>2$

Suppose you have 5 agents in the economy. Then the grand coalition is stable and gives each member a payoff of 2.
Is anything else in the core?
How about (1,2,2,2,3)? (yes)
How about (1,1,2,3,3)? (no)

What would happen to the size of the core if we had *increasing returns to population?* (meaning per captia utility is strictly increasing in population size).
For example
For example, suppose:

$V(n)= n^2$

Here, the core gets large. Do some examples.

Suppose instead that we have *constant returns to population:*

$V(1) = 2n$

Here the core has only one element and equal treatment always prevails.

What would happen to the size of the core if we had *decreasing returns to population?*
For example, suppose:

$V(1) = 1$
$V(2) = 5$
$V(n) = 2n$ for $n > 2$

Suppose you have 10 agents in the economy.
The core is empty! Why? Because smaller coalitions do better than larger ones, the grand coalition can't form.

Let's take a broader view. What if we let a series of smaller coalitions form in the core instead of insisting the agents get together in one big one. This is like allowing local public goods to be provided by local jurisdictions.

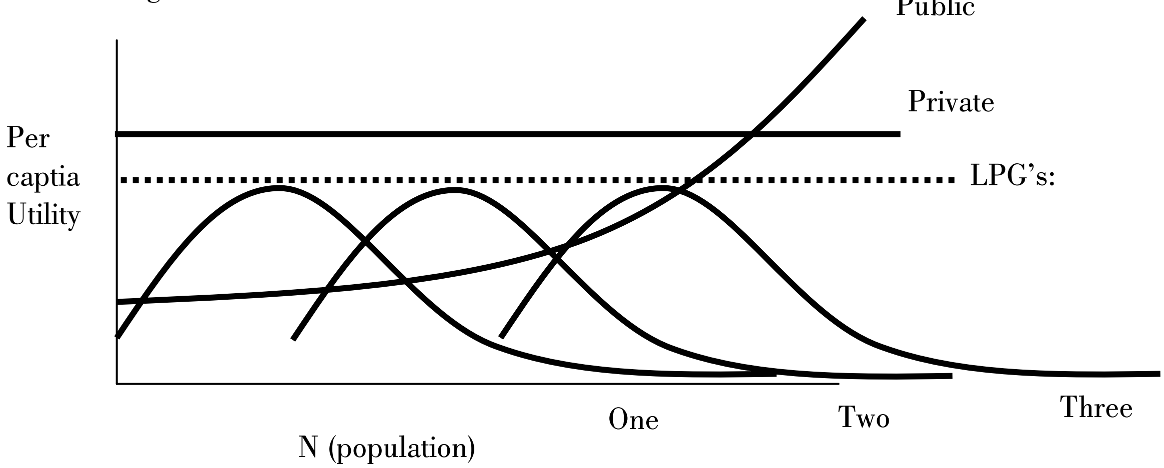
Suppose you have 10 agents in the economy. Then the core consists of five coalitions of two agents each with each agent getting a payoff of 2.5.

Why?
Could a larger coalition form?
Could a smaller coalitions form?
Could the core give agents payoffs that were not equal?
Could the two person coalitions give a payoff or more or less than 2.5?

Thus, the core is unique and also socially optimal.

What would happen if you had nine agents?
(Core would not exist.)
Thus, we get a problem if the population can't be broken up into exactly optimal coalitions without leftovers.
Is this a problem is real life?
No, since in reality people are in motion trying to form optimal coalitions. We do have people floating between groups.

There is a general lesson here.



What would this characteristic function look like in a pure private good exchange economy? (CRPop). This implies the core is unique (in the limit), and satisfies equal treatment. (since agents can form a coalition of half the size and do just as well per capita.)

What about a pure public goods economy? (IRPop). This implies the core gets large

What about a LPG's economy (DRPop)? If agents had to live in one jurisdiction, the core is empty. But observe that the envelope of per captia payoffs as the number Jurisdictions increases proportionally to the population is CRPop. Thus, the core exists when population is an even multiple of optimal jurisdiction size, is unique and satisfies equal treatment

In fact, LPG's economy satisfy something called "small group effectiveness"

What this means is that all of the gains in per capita utility can be realized by groups that are small compared the whole economy. This is a formal way of saying assumption 6 in Tiebout, and is almost a definition of LPG economy.

What about the Tiebout equilibrium? Now we have to think about prices and the commodity space.
What goods are being priced? (n^j, y^j). That is the complete possible set of types of jurisdictions.
We could imagine a price list $p(n^j, y^j)$ for each such club.
To be simple, suppose the prices were the following:

$$p(n^j, y^j) = \frac{y^j}{n^j}$$

Note that these are zero profit prices. Then what would each agent choose?
They would pick socially optimal jurisdictions!

Thus, these prices decentralize the social optimum. A Tiebout equilibrium consists of an allocation, a set of prices and a partition of agents such that given prices, no agent would like to move to a different location, and no city manager can offer an alternative city that would yield positive profits. (We have not talked about the latter, but this is also true for the prices given.)

Note that this means that many goods that have a price will never be purchased.

Examples:

- Cities with property taxes can be seen as admission prices
- Dances with entrance fees
- Movie theaters
- Buses/trains/airplanes

What if we had several different types of agents? Different types would choose different types of jurisdictions, thus, the social optimum and the Tiebout equilibrium are segregated by taste, or taste homogenous.

So far we have only looked at anonymous crowding. All agents are identical to one another.
What if we had:

<u>Differentiated crowding:</u>

Each agent now has two characterizes, taste type and crowding type. As before, the utility function is indexed by t:

$t=1,\ldots,T$

Agents also have a crowding type indexed by c:

$c= 1, \ldots, C$

Examples:

- Smoker/nonsmoker
- Skills/abilities
- Charisma
- Height
- Gender

In general, there are payoff relevant aspects of agents that are exogenous to agents. We care about how many of each type are in our coalition.

There are N_{ct} agents of type ct in the economy. Thus, the population can be described as

$(N_{11}, \ldots, N_{ct}, \ldots, N_{CT}) = N$

Agents form jurisdictions that make up a *partition* of the population. That is, each agent is in one and only one jurisdiction. However, now we care about the crowding type of each agent: A given jurisdiction is described as:

$n^j = (n^j_1, \ldots, n^j_c, \ldots, n^j_c)$

$n^j_c =$ the number of agents of crowding type c in jurisdiction j.

The number of agents in each type in jurisdiction is given by the list:

$n = (n^1, \ldots, n^j, \ldots, n^J)$

Agents care about three things: public goods in the jurisdiction, private goods and the number and crowding type of agents in the jurisdiction:

$U_t(x, y^j, n^j_1, . . ., n^j_c, \ldots n^j_C)$.

Production:

$F(y\ n_1, . . ., n_c, \ldots n_C)$

Thus, having agents with different skills can affect the productivity of a firm or coalition.

Should agents segregated by crowding type? No! examples: a dance, a department, and a firm
Should agent's segregate by taste?
Example: a dance. It would seem that yes, but what if all boys like heavy metal and all girls like N'sync? They should still get together.

<u>What if we have enough agents of each type? Consider the following:</u>

Two crowding types:

Smokers (S)
Nonsmokers (N)

Two taste types:

Lovers of secondhand smoke (L)
Haters of secondhand smoke (H)

Thus, there are four types altogether:

SL, NL, SH, NH

Assume that there are four agents of each type in the population, and agents only get positive utility if they are in coalitions with exactly two members (i.e. consider a matching game and suppress public goods for now).

$$u_H(S,S)=0 \quad u_L(S,S)=10$$
$$u_H(S,N)=5 \quad u_L(S,N)=5$$
$$u_H(N,N)=10 \quad u_L(N,N)=0$$
$$u_H(S,S)\ =0 \quad u_L(S,S)=10$$
$$u_H(S,N)\ =\ 5 \quad u_L(S,N)=5$$
$$u_H(N,N)\ =10 \quad u_L(N,N)=0$$

$$V(SL,\ SL)=V(NH,\ NH)=20$$
$$V(SH,\ NL)=V(SL,\ SH)=V(SL,\ NL)=10$$
$$V(SL,\ NH)=V(SH,\ NH)=V(NL,\ NH)=10$$
$$V(SH,\ SH)=V(NL,\ NL)=0$$

and zero for every other jurisdiction type.

Observe that anything that can be improved upon can be improved upon by two person jurisdictions, and so SGE is satisfied.

A Core State: two (SL, SL), two (NH, NH), and four (SH, NL).

$$U_{SL}=10 \quad U_{NL}=5$$
$$U_{SH}=5 \quad U_{NH}=10.$$

Suppose we tried to taste-homogenize one of the four (SH, NL) jurisdictions.

$$V(SH,\ NL) = 10 = U_{SH} \ \ + U_{NL}$$

but,

$$V(SH,\ \ NH) = 10 < \ 15 = U_{SH} \ \ + U_{NH}.$$

Thus, homogenizing this jurisdiction leads to a loss of utility.

Notice here it is the crowding that causes heterogeneity to be optimal. We notice the following things make it more likely that agents separate by tastes:

1. you care more about public goods
2. You care less about the crowding types of agents
3. You most prefer your own crowding type. (see the above example).

What would prices look like? They would be different in two ways:

$p_c(n^j_1, \ldots, n^j_C,\ \ y^j)$

Obviously, they would have to take into account the crowding profile. Also, they would have to change different agent different prices accounting their own crowding type.

Examples:

- College admissions
- Jobs
- Housing (no)
- Dances?

One thing that is assumed in the above is that crowding is exogenous. However, many things that affect others are chosen. Let now consider a simple case of

Endogenous crowding type:

To the above, we simply add and "educational cost functions"

$$E(c) = x$$

This gives the cost in terms of private good of becoming crowding type c. Think of this of the cost as tuition and the crowding type as a skill. It might cost more to become a doctor than an accountant. It takes effort to learn a new language or to dance well. Now we can choose what to be.

Clearly this does not apply to crowding characteristics such as gender.

Notice the following:

The admission price function $p_c(n^j_1, . . ., n^j_C, y^j)$ has to take a special form. The difference in the price for any two crowding types c, c' is exactly the difference between the cost of the becoming these types. Otherwise there is an arbitrage.

Also, note that agent's would never opt to become a crowding type whose externality they hated, No one would choose to be a smoking hater. All haters would choose to be nonsmokers and all lovers, smokers.

Finally, since agents can be anything they want, agents will now segregate by tastes so that they get their most preferred public goods bundle, and then distribute themselves over crowding types to get the optimal crowding profile. Again, note that in equilibrium, agents will be indifferent over which crowding type they become. Therefore, optimal coalitions/ jurisdictions are *taste homogeneous.*

Endogenous crowding type with genetic differences.

This model may be too simple as it assumes that agents are equally able to acquire different crowding characteristics. However, in real life, people have different types of innate abilities. These abilities themselves do not generate crowding externalities, but they affect the cost of acquiring different abilities. Formally. Agents now have two basic characteristics, tastes and genetic types:

$t = 1, ..., T$
$g = 1, ..., G$

Examples of genetic types:

- Intelligence
- Athletic ability
- Artistic talent
- Memory

The educational cost function now depends on this:

$E(c, g) = x$

Thus smart guys might find is less costly than dumb ones get an MD, and dumb guys might find it easier to get MBA's than smart ones.

We can also imagine that people care about the crowding type they express. Formally, this means that we must extend the utility function:

$U_t(x, y^j, c, n^j_1, \ldots, n^j_c, \ldots n^j_C)$.

Note that we don't care about the genetic types, but about how agents have chosen to express these in crowding types.

It might be impossible to for people with certain g's to become certain c's. For example, with my endowment, I chose to be an economist. No matter how much I might have wished it, I could never have become Julio Iglesias.

Prices continue to take the form:

$p_c(n^j_1, \ldots, n^j_C, y^j)$

Note that only crowding types (which convey externalities) are priced, not generic types.

These admissions prices may be positive or negative depending of the scarcity of various agents.

As a special case, we can also think about this as a model of skill choice and firm formations. The price can be in effect a wage of a person with certain skills. The point is that we can use this type of model to think about how a wide range of coalition are formed

Putting this all together, one now chooses crowding type thinking about three things. First, how much does one want to be a given crowding types (I really want to be an artist). Second how good on is at acquired that particular skill (I can't draw a straight line, but I am very good at singing) Third, what admission price is paid to different crowding types affects the choice (but I am also fairly good at math, and even though I would not really enjoy it, being an accountant would pay very well.)

Note that in this case, optimal collations may not be taste homogenous. For example, what is an optimal marriage? Should your spouse share your tastes? One might think yes. If she likes Chinese food, and I like Mexican we will have a hard time going to dinner together. Can a marriage last if she's a little bit country and I'm a little bit rock and roll?

Clearly there are some reasons to share the same tastes. Where might this not be true?

Consider the public good called "babies" which are jointly produced by contributions of each partner. It turns out that I like to stay up late while my wife likes to go to bed early. This makes it relatively less costly to become the crowding type "giver of late night feedings" and relatively less costly for my wife to become the crowding type "giver of early morning feedings". My sister likes to do home repair, while my brother-in-law likes to do financial planning. They have each devoted time to learning how to do each job. Thus, there is a gain from trade if you partner with someone who is either good at, or who likes to do jobs that you are poor at or don't like.

Should you want your spouse to work out and get in shape? Get a raise? Go back to school? Applications to work environments?

How do smart people marry dumb people? Why or why not? Prices? Preferences? Generally what causes people to match. You have to bring something to the party.

What's love got to do with it?

<u>Variable usage</u>

So far we have only made membership in a club relevant. In real life, we may care about the extent of participation. For example, we might join a county club first of all for the membership (crowding profile). After all, being a member of the right club pays dividends. However, a secondary reason might be to play golf. Here we are crowded negatively by the total number of visits that other agents make to course (V). We also care about how many visits we make to the course (v^i). Thus:

$U_t(x, y^j, n^j_1, \ldots, n^j_c, \ldots n^j_C, v^i, V)$.

Prices might now be two-part. First there might be fixed joining price as above that depends on the qualities of the club:

$p_c(n^j_1, \ldots, n^j_C, y^j, V)$

There might also be a per visit price (greens fees):

$q_c(n^j_1, \ldots, n^j_C, y^j, V)$

Note that the to price usage efficiently, the usage fee should equal the marginal cost. What is the marginal cost? Two parts. First, it might cost something to provide the use on the margin (examples: caddies). Sometimes this cost is nothing, it only costs to provide the public facility (example: roads). Second, it might cost others because of congestions (V is often a negative in utility (example: roads a rush hour)).

How should I price the use of a bridge?
Should this price vary with time of day?
Is a school a variable usage club? Are visits to class priced? Why or why not?

Obviously, usage congestions could be anonymous or depend on crowing types. Crowding type could be endogenous or exogenous, depend on genetic type or not.

Some of the active research areas in public economic theory

Stock externalities
Transboundary pollution
Global warming
Environmental Kuznets Curve

Voluntary contribution public good models
Charity and altruism
Education Finance and reform

Fiscal competition
Coalitions and clubs
Local public goods
Urban economics and regional science
Fiscal federalism

Tax evasion
Corruption

Social norms and social cohesion
Social networks

Equality and inequity
Social choice
Social security and social insurance
Growth policy
Terrorism
Immigration
Political economy
Voting

Tools that have always been in use:

Comparative statics
General equilibrium theory
Overlapping generation's models
Dynamic optimization approaches.
Cooperative game theory (Core, social choice, bargaining theory)
Noncooperative game theory (mechanism design, political economy)

Tools that are gaining prominence:

Evolutionary Game theory
Network theory
Simulations and numerical approaches
Behavioral approaches
Experimental approaches.