Nonconvex bargaining problems

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March 2004

Abstract

This paper studies compact and comprehensive bargaining problems for n players and axiomatically characterize the extensions of the three classical bargaining solutions to nonconvex bargaining problems: the Nash solution, the egalitarian solution and the Kalai-Smorodinsky solution. Our characterizing axioms are various extensions of Nash's original axioms.

1 Introduction

This paper considers *nonconvex* bargaining problems for n players. Specifically, we study (normalized) bargaining problems that are *compact* and *comprehensive*, but are not necessarily convex. Nonconvex bargaining problems can arise in many economic contexts when, for example, individuals are non-expected utility maximizers. They also arise naturally in bargaining problems when individuals are not characterized by their utilities but by their *capability sets à la* Sen (1985) (see Xu and Yoshihara (2004) for such cases).

The literature has some discussions on nonconvex bargaining problems. For example, there exists a number of characterizations of the Nash bargaining solution for the class of compact and comprehensive bargaining problems. However, in all the characterization results, either a type of continuity property is imposed (see, for example, Kaneko (1980), Herrero (1989), Conley and Wilkie (1996)), or the class of bargaining problems contains finite bargaining problems in addition to those that are compact and comprehensive (see, for example, Mariotti (1999)). The purpose of this paper is two-fold. First, we give a new characterization of the Nash bargaining solution for the class of compact and comprehensive bargaining problems by four axioms: Efficiency, Symmetry, Scale Invariance and Contraction Independence, and provide a simple proof that highlights the crucial role that Contraction Independence plays. Because of our proof method, it is interesting to note that we do not use any continuity type axiom in our characterization. The four axioms used in the characterization result of the Nash solution are natural extensions of Nash's original four axioms (Nash (1950)) in our context. Viewed in this way, this characterization result reported in the paper is perhaps closer to Nash's original program than those already existing in the literature. Secondly, we use variants of the four axioms used for characterizing the Nash solution to characterize the egalitarian solution (Kalai (1977)) and the Kalai-Smorodinsky solution (Kalai and Smorodinsky (1975)) for nonconvex bargaining problems. Our characterization results of the egalitarian and the Kalai-Smorodinsky solutions again highlights the crucial role that Contraction Independence or Weak Contraction Independence (see Section 3 for the formal definition) plays. It should be noted that our characterizations of the egalitarian as well as the Kalai-Smorodinsky solutions do not use the commonly used Monotonicity type axioms for characterizations of those two solutions.

The remainder of the paper is organized as follows. In Section 2, we lay down some basic notation and definitions. Section 3 presents our axioms and their discussions. The main results and their proofs are contained in Section 4. We conclude the paper with a few remarks in Section 5 comparing and contrasting the axioms used in characterizing the three solutions.

2 Notation and Definitions

$\mathbf{R}_+$ is the set of all non-negative real numbers and $\mathbf{R}_{++}$ is the set of all positive numbers. $\mathbf{R}^n_+$ (resp. $\mathbf{R}^n_{++}$) is the n-fold Cartesian product of $\mathbf{R}_+$ (resp. $\mathbf{R}_{++}$). For any $x, y \in \mathbf{R}^n_+$, we write $x > y$ to mean $[x_i \geq y_i$ for all $i \in N$ and $x \neq y]$, and $x \gg y$ to mean $[x_i > y_i$ for all $i \in N]$. For any $x \in \mathbf{R}^n_+$ and any non-negative number α, we write $z = (\alpha; \mathbf{x}_{-i}) \in \mathbf{R}^n_+$ to mean that $z_i = \alpha$ and $z_j = x_j$ for all $j \in N \setminus \{i\}$. A subset $A \subseteq \mathbf{R}^n_+$ is said to be *non-trivial* if there exists $a \in A$ such that $a \gg 0$. Let Σ be the set of all non-trivial, compact and comprehensive subsets of $\mathbf{R}^n_+$. Elements in Σ are interpreted as (normalized) bargaining problems. A bargaining solution F assigns a nonempty subset $F(A)$ of A for every bargaining problem $A \in \Sigma$.

Let π be a permutation of N. The set of all permutations of N is denoted by Π. For all $x = (x_i)_{i \in N} \in \mathbf{R}^n_+$, let $\pi(x) = (x_{\pi(i)})_{i \in N}$. For all $A \in \Sigma$ and any permutation $\pi \in \Pi$, let $\pi(A) = \{\pi(a) : a \in A\}$. For any $A \in \Sigma$, we say that A is *symmetric* if $A = \pi(A)$ for all $\pi \in \Pi$.

For all $A \in \Sigma$ and all $i \in N$, let $m_i(A) = \max\{a_i : (a_1, \cdots, a_i, \cdots, a_n) \in A\}$. Therefore, $m(A) \equiv (m_i(A))_{i \in N}$ is the *ideal point* of A. For all $A \subseteq \mathbf{R}^n_+$, define the *comprehensive hull of* A, to be denoted by *compA*, as follows:

$$compA \equiv \Big\{ z \in \mathbf{R}^n_+ : z \leq x \text{ for some } x \in A \Big\}.$$

Definition 1: A bargaining solution F over Σ is the Nash solution if for all $A \in \Sigma$, $F(A) = \{a \in A : \prod_{i \in N} a_i \geq \prod_{i \in N} x_i$ for all $x \in A\}$.

Definition 2: A bargaining solution F over Σ is the egalitarian solution if for all $A \in \Sigma$, $F(A) = \{a \in A : a_i = a_j$ for all $i, j \in N$ and there is no $x \in A$ such that $x >> a\}$.

Definition 3: A bargaining solution F over Σ is the Kalai-Smorodinsky solution if for all $A \in \Sigma$, $F(A) = \{a \in A : m_i(A)/a_i = m_j(A)/a_j$ for all $i, j \in N$ and there is no $x \in A$ such that $x >> a\}$.

Our notion of the Nash solution for nonconvex bargaining problems is identical to the one proposed by Kaneko (1980).[1] It should be noted that, given that Σ contains all non-trivial, compact and comprehensive bargaining problems, for any $A \in \Sigma$, the Nash solution $F(A)$ can contain more than one alternative, while both the egalitarian and the Kalai-Smorodinsky solutions are singletons.

3 Axioms

In this section, we present our axioms that are to be used for characterization results. We start with two efficiency type axioms which are commonly invoked in the literature.

Efficiency (E): For any $A \in \Sigma$ and any $a \in F(A)$, there is no $x \in A$ such that $x > a$.

Weak Efficiency (WE): For any $A \in \Sigma$ and any $a \in F(A)$, there is no $x \in A$ such that $x \gg a$.

The next two axioms are natural generalizations of Nash's original symmetry axiom in our context.

Symmetry (S): For any $A \in \Sigma$, if A is symmetric, then $[a \in F(A) \Rightarrow \pi(a) \in F(A)$ for all $\pi \in \Pi]$.

Strong Symmetry (SS): For any $A \in \Sigma$, if A is symmetric, then $[a \in F(A) \Rightarrow a_i = a_{\pi(i)}$ for all $i \in N$ and all $\pi \in \Pi]$.

Symmetry is a natural generalization of Nash's original symmetry axiom to nonconvex problems and is also discussed in Mariotti (1999). Strong Symmetry is a stronger requirement than Symmetry. It should be noted that, when restricted to convex bargaining problems, and bargaining solutions are

[1]Mariotti (1999) also discusses axiomatic characterization of the Kaneko type of the Nash solution for nonconvex problems, although his domain is larger than ours in the sense that it includes "*finite bargaining problems.*" In contrast, Herrero's proposal (Herrero (1989)) for the Nash extension solution constitutes a *superset* of the set of the Kaneko type solution outcomes on each nonconvex problem, and Conley and Wilkie (1996) proposes an extension of the Nash solution which is a single-valued mapping in that domain.

required to be single-valued mappings, the two symmetry axioms coincide with and are identical to Nash's original Symmetry axiom.

The next axiom is the familiar scale invariance property commonly used in both convex (see, for example, Nash (1950)) and nonconvex bargaining problems (see, for example, Conley and Wilkie (1996), Herrero (1989), Mariotti (1999).

Scale Invariance (SI): For all $A \in \Sigma$ and all $\alpha \in \mathbf{R}^n_{++}$, if $\alpha A = \{(\alpha_i a_i)_{i \in N} : a \in A\}$ then $F(\alpha A) = \{(\alpha_i a_i)_{i \in N} : a \in F(A)\}$.

The final two axioms are extensions of Nash's original Independence of Irrelevant Alternatives.

Contraction Independence (CI): For any $A, B \in \Sigma$, if $B \subseteq A$ and $B \cap F(A) \neq \emptyset$, then $F(B) = B \cap F(A)$.

Weak Contraction Independence (WCI): For any $A, B \in \Sigma$, if $m(A) = m(B)$, $B \subseteq A$ and $B \cap F(A) \neq \emptyset$, then $F(B) = B \cap F(A)$.

Contraction Independence has been widely used in the literature of nonconvex bargaining problems. Weak Contraction Independence is new and is weaker than Contraction Independence: it restricts contractions to those problems that have the same ideal point.

4 Extensions of the classical bargaining solutions and their characterizations

In this section, we provide axiomatic characterizations of the Nash solution, the egalitarian solution and the Kalai-Smorodinsky solution.

Theorem 1: A bargaining solution F over Σ is the Nash solution if and only if it satisfies Efficiency, Symmetry, Scale Invariance and Contraction Independence.

Proof. It can be checked that if F is the Nash solution over Σ then it satisfies the four axioms in Theorem 1. Thus, we need only to show that if a bargaining solution F over Σ satisfies Efficiency, Symmetry, Scale Invariance and Contraction Independence, then it must be the Nash solution.

Let F over Σ satisfy the above four axioms. Given any bargaining problem $A \in \Sigma$, we first show that

> **Claim 1:** for any x and a which are both efficient in A, and if $x \in A$ is such that $\prod_{i \in N} x_i < \prod_{i \in N} a_i$, then $x \notin F(A)$.

Let $a, x \in A$ be such that they are efficient in A and that $\prod_{i \in N} x_i < \prod_{i \in N} a_i$. Suppose to the contrary that

$$x \in F(A).$$

Consider the bargaining problem $B = \text{comp}\{x, a\}$. From the construction, $B \subseteq A$. By Contraction Independence,

$$x \in F(B).$$

Let $z \in \mathbf{R}^n_+$ be $(z_1; \mathbf{x}_{-1}) \in \mathbf{R}^n_+$ where z_1 is such that $z_1 \prod_{i=2}^n x_i = \prod_{i \in N} a_i$. Since $\prod_{i \in N} x_i < \prod_{i \in N} a_i$, we must have $z_1 > x_1$. Consider the bargaining problem $C \in \Sigma$, which is defined as:

$$C = \text{ comp } \{a, z\}.$$

Clearly, $B \subseteq C$. By choosing $\alpha \in \mathbf{R}^n_{++}$ appropriately, given that $z_1 \prod_{i=2}^n x_i = \prod_{i \in N} a_i$, we can have $(\alpha_i z_i)_{i \in N} = \pi^0((\alpha_i a_i)_{i \in N})$ and $(\alpha_i a_i)_{i \in N} = \pi^0((\alpha_i z_i)_{i \in N})$ for some permutation $\pi^0 \in \Pi$. Let $D = \{y \in \mathbf{R}^n_+ : y = (\alpha_i b_i)_{i \in N}$ for all $b \in C\}$. Note that a and z are the two and only two efficient points in C. It then follows that $(\alpha_i a_i)_{i \in N}$ and $(\alpha_i z_i)_{i \in N}$ are the two and only two efficient points in D. Construct the following bargaining problem $G \in \Sigma$:

$$G \equiv \cup_{\pi \in \Pi} \pi(D).$$

Clearly, G is symmetric. By Symmetry and Efficiency, it must be true that

$$\{(\alpha_i a_i)_{i \in N}, (\alpha_i z_i)_{i \in N}\} \subseteq F(G).$$

Contraction Independence now implies that

$$\{(\alpha_i a_i)_{i \in N}, (\alpha_i z_i)_{i \in N}\} = F(D).$$

By Scale Invariance,

$$\{a, z\} = F(C).$$

Noting that $B \subseteq C$, by Contraction Independence, it follows that

$$\{a\} = F(B)$$

a contradiction. Therefore, $x \notin F(A)$. Claim 1 is thus proved. Therefore, we must have the following:

$$\text{for any } A \in \Sigma, F(A) \subseteq \{a \in A : \prod_{i \in N} a_i \geq \prod_{i \in N} x_i \ \forall x \in A\}.$$

It remains to show that for any $A \in \Sigma$, for all $z \in \{a \in A : \prod_{i \in N} a_i \geq \prod_{i \in N} x_i \ \forall x \in A\}$, it must be true that $z \in F(A)$. Given that $F(A)$ is not empty, from the above, let $a \in F(A)$. Then, a must be such that $\prod_{i \in N} a_i \geq \prod_{i \in N} x_i$ for all $x \in A$. Suppose there exists $y \in A$ such that $\prod_{i \in N} y_i = \prod_{i \in N} a_i$ and yet $y \notin F(A)$. Consider the following bargaining problem: $X \equiv \text{comp}\{a, y\}$. Note that $X \subseteq A$. Since $a \in F(A)$ and $y \notin F(A)$, by Contraction Independence, we must have $a \in F(X)$ and $y \notin F(X)$. By appropriately choosing $\alpha \in \mathbf{R}^n_{++}$, given that $\prod_{i \in N} y_i = \prod_{i \in N} a_i$, we can have $\pi^0((\alpha_i a_i)_{i \in N}) = (\alpha_i y_i)_{i \in N}$ and $\pi^0((\alpha_i y_i)_{i \in N}) = (\alpha_i a_i)_{i \in N}$ for some permutation π^0 over N. Consider $X' \equiv \{(\alpha_i x_i)_{i \in N} : (x_i)_{i \in N} \in X\}$. By Scale Invariance,

$$(\alpha_i a_i)_{i \in N} \in F(X') \text{ and } (\alpha_i y_i)_{i \in N} \notin F(X').$$

Now, construct the bargaining problem $Z \equiv \cup_{\pi \in \Pi} \pi(X')$. Clearly, Z is symmetric and $X' \subseteq Z$. By Symmetry and Efficiency, it must be true that

$$\{(\alpha_i a_i)_{i \in N}, (\alpha_i y_i)_{i \in N}\} \subseteq F(Z).$$

Noting that $X' \subseteq Z$, by Contraction Independence, it follows that

$$\{(\alpha_i a_i)_{i \in N}, (\alpha_i y_i)_{i \in N}\} = F(X').$$

Scale Invariance now implies that

$$\{a, y\} = F(X),$$

a contradiction. Therefore, $y \in F(A)$. That is, we have shown that

$$\text{for any } A \in \Sigma, \{a \in A : \prod_{i \in N} a_i \geq \prod_{i \in N} x_i \ \forall x \in A\} \subseteq F(A).$$

Therefore,

$$\text{for any } A \in \Sigma, F(A) = \{a \in A : \prod_{i \in N} a_i \geq \prod_{i \in N} x_i \ \forall x \in A\}.$$

$\diamond$

Theorem 2: A bargaining solution F over Σ is the egalitarian solution if and only if it satisfies Weak Efficiency, Strong Symmetry and Contraction Independence.

Proof. It can be checked that if F is the egalitarian solution over Σ then it satisfies the four axioms in Theorem 2. Thus, we need only to show that if a bargaining solution F over Σ satisfies Weak Efficiency, Strong Symmetry and Contraction Independence, then it must be the egalitarian solution.

Let F over Σ satisfy the above three axioms. Given any bargaining problem $A \in \Sigma$, we first show that

> **Claim 2:** for any x and a which are weakly efficient in A, and if $x, a \in A$ is such that $[x_i \neq x_j$ for some $i, j \in N]$, and $[a_i = a_j$ for all $i, j \in N]$, then $x \notin F(A)$.

Let $a, x \in A$ be such that they are weakly efficient and that $[x_i \neq x_j$ for some $i, j \in N]$, and $[a_i = a_j$ for all $i, j \in N]$. Suppose to the contrary that

$$x \in F(A).$$

Consider the bargaining problem

$$B = \text{comp}\{a, x\}.$$

Note that $B \subseteq A$. By Contraction Independence,

$$x \in F(B).$$

Consider the bargaining problem $C \in \Sigma$, which is defined below:

$$C = \cup_{\pi \in \Pi} \pi(B).$$

By construction, C is symmetric, $a \in C$, a is weakly efficient in C, and $B \subseteq C$. By Strong Symmetry and Weak Efficiency, it then follows that $F(C) = \{a\}$. Noting that $B \subseteq C$, by Contraction Independence, it follows that

$$F(B) = \{a\}$$

a contraction with $x \in F(B)$. Therefore, $x \notin F(A)$. Then, by Weak Efficiency, it must be true that

$$F(A) = \{a \in A : a_i = a_j \ \forall i \in N \text{ and there exists no } x \in A \text{ such that } x >> a\}.$$

$\diamond$

Theorem 3: A bargaining solution F over Σ is the Kalai-Smorodinsky solution if and only if it satisfies Weak Efficiency, Strong Symmetry, Scale Invariance and Weak Contraction Independence.

Proof. It can be checked that if F is the Kalai-Smorodinsky solution over Σ then it satisfies the four axioms in Theorem 3. Thus, we need only to show that if a bargaining solution F over Σ satisfies Weak Efficiency, Strong Symmetry, Scale Invariance and Weak Contraction Independence, then it must be the Kalai-Smorodinsky solution.

Let F over Σ satisfy the above four axioms. Given any bargaining problem $A \in \Sigma$, by Scale Invariance, without loss of generality, we take that $[m_i(A) = m_j(A)$ for all $i, j \in N]$. We need to show that if a is weakly efficient in A and $[a_i = a_j$ for all $i, j \in N]$, then $F(A) = \{a\}$.

Let $a \in A$ be such that it is weakly efficient and that $[a_i = a_j$ for all $i, j \in N]$. Consider the bargaining problem

$$B \equiv \text{ comp}\{a, (m_1(A); a_{-1}), \cdots, (m_i(A); a_{-i}), \cdots, (m_n(A); a_{-n})\}.$$

From the construction, B is symmetric and $A \subseteq B$. By Weak Efficiency and Strong Symmetry, we must have $F(B) = \{a\}$. Noting that $A \subseteq B$ and $m(A) = m(B)$, by Weak Contraction Independence, we then obtain $F(A) = \{a\}$. Therefore, Theorem 3 is proved. $\diamond$

To conclude this section, we make the following observations concerning the logical independence of the axioms used in each of the above three theorems.

Proposition 1: The axioms Efficiency, Symmetry, Scale Invariance and Contraction Independence are logically independent.

Proof. Consider the following solutions:

(1) For all $A \in \Sigma$, $F_1(A) = A$;

(2) For all $A \in \Sigma$, $F_2(A) = \{a \in A : a_1^2 \prod_{i=2}^n a_i \geq x_1^2 \prod_{i=2}^n x_i$ for all $x \in A\}$;

(3) For all $A \in \Sigma$, $F_3(A) = \{a \in A : \sum_{i \in N} a_i \geq \sum_{i \in N} x_i$ for all $x \in A\}$;

(4) For all $A \in \Sigma$, $F_4(A) = \{a \in A : a$ is efficient $\}$.

It can be checked that F_1 satisfies Symmetry, Scale Invariance and Contraction Independence while violates Efficiency; F_2 satisfies Efficiency, Scale Invariance and Contraction Independence while violates Symmetry; F_3 satisfies Efficiency, Symmetry and Contraction Independence while violates Scale Invariance; and F_4 satisfies Efficiency, Symmetry and Scale Invariance while violates Contraction Independence. $\diamond$

Proposition 2: The axioms, Weak Efficiency, Strong Symmetry and Contraction Independence are logically independent.

Proof. Consider the following solutions:

(5) For all $A \in \Sigma$, $F_5(A) = \{a \in A : a_i = a_j$ for all $i, j \in N\}$;

(6) For all $A \in \Sigma$, if A is symmetric then $F_6(A)$ is given by the egalitarian solution; otherwise, $F_6(A)$ is given by the Nash solution.

Clearly, F_5 satisfies Strong Symmetry and Contraction Independence while violates Weak Efficiency; the Nash solution satisfies Weak Efficiency and Contraction Independence while violates Strong Symmetry; and F_6 satisfies Weak Efficiency and Strong Symmetry while violates Contraction Independence. $\diamond$

Proposition 3: The axioms Weak Efficiency, Strong Symmetry, Scale Invariance and Weak Contraction Independence are logically independent.

Proof. Consider the following solutions:

(7) For all $A \in \Sigma$, $F_7(A) = \{a \in A : a_i/m_i(A) = a_j/m_j(A)$ for all $i, j \in N\}$;

(8) For all $A \in \Sigma$, if A is such that $m_i(A) = m_j(A)$ for all $i, j \in N$, then $F_8(A)$ is given by the Kalai-Smorodinsky solution; otherwise, $F_8(A)$ is given by $\{a \in A : a$ is weakly efficient in $A\}$;

(9) Let $\Sigma^* = \{A \in \Sigma : m_i(A) = m_j(A)$ for all $i, j \in N\}$. Define F_9 as follows: for all $A \in \Sigma^*$, if A is symmetric, then $F_9(A)$ is given by the Kalai-Smorodinsky solution, and if A is not symmetric, then $F_9(A) = \{a \in A : a$ is weakly efficient in $A\}$; for all $A \in \Sigma \backslash \Sigma^*$, $F_9(A) = \{(a_i^*/\alpha_i)_{i \in N} : a^* \in F(A^*)\}$ where $A^* = \alpha A = \{(\alpha_i a_i)_{i \in N} : a \in A\}$ for some $\alpha \in \mathbf{R}^n_{++}$ such that $A^* \in \Sigma^*$.

It can be checked that F_7 satisfies Strong Symmetry, Scale Invariance and Weak Contraction Independence while violates Weak Efficiency; the Nash solution satisfies Weak Efficiency, Scale Invariance and Contraction Independence while violates Strong Symmetry; F_8 satisfies Weak Efficiency, Strong Symmetry and Weak Contraction Independence while violates Scale Invariance; and F_9 satisfies Weak Efficiency, Strong Symmetry and Scale Invariance while violates Weak Contraction Independence. $\diamond$

5 Conclusion

In this paper, we have presented a unified framework to provide axiomatic characterizations of the extensions of the three classical bargaining solutions for nonconvex bargaining problems. Our characterizations are simpler than those existing in the literature. Our axioms are various natural generalizations of the axioms used in Nash's original discussion of the bargaining problems for convex bargaining problems. The following table summarizes our findings:

Table 1

Axioms\ Solutions	NS	ES	KS
(E)	$\oplus$	$\times$	$\times$
(WE)	$\bigcirc$	$\oplus$	$\oplus$
(S)	$\oplus$	$\bigcirc$	$\bigcirc$
(SS)	$\times$	$\oplus$	$\oplus$
(SI)	$\oplus$	$\times$	$\oplus$
(CI)	$\oplus$	$\oplus$	$\times$
(WCI)	$\bigcirc$	$\bigcirc$	$\oplus$

where

NS is for Nash Solution, ES for Egalitarian Solution, and KS for Kalai-Smorodinsky Solution

$\oplus$ stands for that the axiom is used for the characterization,

$\bigcirc$ stands for that the axiom is satisfied by the solution,

$\times$ stands for that the axiom is violated by the solution.

Clearly, Weak Efficiency, Symmetry and Weak Contraction Independence are satisfied by all three solutions. It is also clear that the Nash solution satisfies all the axioms but Strong Symmetry, the egalitarian solution satisfies all the axioms but Efficiency and Scale Invariance, and the Kalai-Smorodinsky solution satisfies all but Efficiency and Contraction Independence. Note that Theorem 2 (*resp.* Theorem 3) constitutes a strengthening of the characterization of the egalitarian solution (*resp.* the Kalai-Smorodinsky solution) by Conley and Wilkie (1991), since Contraction Independence (*resp.* Weak Contraction Independence) is logically implied by the monotonicity axiom (*resp.* the *weak monotonicity* axiom) of Kalai (1977) in the presence of Weak Efficiency.

It is also interesting to note that the Kalai-Smorodinsky solution has some constrained contraction property. It implies that once a partition of the class of bargaining problems is defined, where each equivalence class of this partition consists of the bargaining problems with the same ideal point, then the Kalai-Smorodinsky solution is rationalizable within each equivalence class of the problems. This fact gives us some new insight on the rational choice property of this solution, which the previous literature does not provide since it is widely considered that it has no rational choice property.

It is hoped that our characterizations will shed some new lights on the three solutions for nonconvex bargaining problems.

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