Illiquidity Component of Credit Risk*

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Abstract

We describe and contrast three different measures of an institution's credit risk. "Insolvency risk" is the conditional probability of default due to deterioration of asset quality if there is no run by short term creditors. "Total credit risk" is the unconditional probability of default, either because of a (short term) creditor run or (long run) asset insolvency. "Illiquidity risk" is the difference between the two, i.e., the probability of a default due to a run when the institution would otherwise have been solvent. We discuss how the three kinds of risk vary with balance sheet composition. Illiquidity risk is (i) decreasing in the "liquidity ratio" – the ratio of realizable cash on the balance sheet to short term liabilities; (ii) decreasing in excess return to debt; and (iii) increasing in the solvency uncertainty – a measure of ex post variance of the asset portfolio. Increasing the liquidity ratio has decreasing returns to reducing illiquidity risk.

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1 Introduction

Credit risk refers to the risk of default by borrowers. In the simplest case, where the term of the loan is identical to the term of the borrower's cash flow, credit risk arises from the uncertainty over the cash flow from the borrower's project. However the turmoil in credit markets in the financial crisis that erupted in 2007 once again highlighted the limitations of focusing just on the value of the asset side of banks' balance sheets. The problem can be posed most starkly for institutions such as Bear Stearns or Lehman Brothers that financed themselves through a combination of short-term and long-term debt, but where the heavy use of short-term debt made the institution vulnerable to a run by the short term creditors.

The issue is highlighted in an open letter written by Christopher Cox, the (then) chairman of the U.S. Securities and Exchange Commission (SEC) explaining the background and circumstances of the run on Bear Stearns in March 2008.[1]

> "[T]he fate of Bear Stearns was the result of a lack of confidence, not a lack of capital. When the tumult began last week, and at all times until its agreement to be acquired by JP Morgan Chase during the weekend, the firm had a capital cushion well above what is required to meet supervisory standards calculated using the Basel II standard.
>
> Specifically, even at the time of its sale on Sunday, Bear Stearns' capital, and its broker-dealers' capital, exceeded supervisory standards. Counterparty withdrawals and credit denials, resulting in a loss of liquidity – not inadequate capital – caused Bear's demise."

Thus, in spite of Bear Stearns meeting the letter of its regulatory capital requirements, it got into trouble because its lenders stopped lending. The implication is that the run was liquidity-based rather than solvency-based.

The idea that self-fulfilling bank runs are possible is well established in the banking literature (see Bryant (1980) and Diamond and Dybvig (1983)).[2]

[1] Letter to the Chairman of the Basel Committee on Banking Supervision, dated March 20th 2008, posted on the SEC website on: http://www.sec.gov/news/press/2008/2008-48.htm

[2] See Gorton (2010) for a modern variation on the classic bank run scenario with an account of the crisis of 2007 as a banking panic with a run on repos rather than deposits.

But the sharp distinction between solvency and liquidity in the SEC Chairman's letter may not be so easy to draw in practice, even ex post. Our current understanding of the relation between insolvency risk and illiquidity risk is not well developed. Existing models tend to focus on one or the other and not on the interaction between the two. We regard this division of attention as untenable. Runs don't happen out of the blue; they tend to occur when there are also concerns about the quality of the assets, as in the case of Bear Stearns in 2008 and as documented by Gorton (1988) for U.S. bank runs during the 1863-1914 National Banking Era. It is sometimes difficult to tell (even ex post) whether the run merely hastened the failure of a fundamentally insolvent bank, or whether the run scuppered an otherwise sound institution. Nevertheless, the distinction between insolvency and illiquidity is meaningful as a counterfactual proposition asking what *would* have happened in unrealized states of the world. The distinction is also important for the policy choices, since the policy response will depend on whether the bank is fundamentally solvent or not. A solvent but illiquid bank could be given emergency funding to tide it over the crisis, but an insolvent bank is best dealt through least cost resolution. In order to address counterfactual "what if" questions, a theoretical framework is required.

For the ex ante pricing of total credit risk, it is important to take account of the probability of a run. This is both because the occurrence of a run will undermine the debt value, and because a run will tend to destroy recovery values through disorderly liquidation under distressed circumstances. Merely focusing on the credit risk associated with the fundamentals of the assets will underestimate the total credit risk faced by a long term creditor. In what follows, we describe a framework that can be used to address these questions. A leveraged financial institution funds its assets using short- and long-term debt, as well as its own equity. We show how global game methods (introduced by Carlsson and van Damme (1993) and used in Morris and Shin (1998, 2003)) can solve for the unique equilibrium in the roll-over game among short-term creditors. In particular, we provide an accounting framework to decompose total credit risk into its components. First, the eventual asset value realization may be too low to pay off all debt; this is the insolvency component of credit risk, or "insolvency risk". Second, a run by the short-term creditors may precipitate the failure of the institution even though, in the absence of the run, the asset realizations would have been high enough to pay all creditors; this is the illiquidity component of credit risk, or "illiquidity risk". We illustrate how total credit risk can be decomposed into insolvency risk and illiquidity risk, and how the two are jointly determined as a function of the underlying balance sheet.

Our analysis can be used to provide comparative statics and thus policy analysis.[3] Illiquidity risk is decreasing in the "liquidity ratio" – the ratio of realizable cash on the balance sheet to short term liabilities. This gives the basic rationale for liquidity regulation. A less obvious comparative static is that illiquidity risk is more responsive to changes in the liquidity ratio when the level of liquidity is low. Thus there are decreasing returns to liquidity regulation. This is because the value of preventing runs is lower when the bank is more likely to fail because of insolvency. Illiquidity risk is increasing (and in fact linear) in solvency uncertainty, as measured by the standard deviation of ex post returns. The conclusion that there is no illiquidity risk without solvency uncertainty is an intuitive one, but not present in many models and much discussion of illiquidity risk. Illiquidity risk is also decreasing in the excess return offered to short term creditors over outside their outside options.

Our benchmark analysis focusses on bank failure due to illiquidity when the bank cannot payoff creditors, or "run risk". Another reason for bank failure is "fire sale risk": the bank can pay off short term creditors but - when it does so - there is degradation of the balance sheet. This degradation then leads to later failure. We can thus further decompose illiquidity risk these two components: run risk is the probability of failure through inability to pay off short term creditors, even though the bank would have been solvent in the absence of a run; fire sale risk is the probability of failure of the bank via balance sheet degradation, where there the bank survives the withdrawal of short term borrowing and would have been solvent in the absence of the balance sheet degradation. We make two points about such fire sale risk. First, we note that run risk in our model or any other model can always be interpreted as taking the form of fire sale risk alone: suppose that the bank never fails due to short term withdrawals per se, but that even if the bank "survives" the run, withdrawals above a certain level will cause the bank to become a "zombie" bank" that is guaranteed to fail at a later date. But we also consider how we can decompose illiquidity risk into run risk and fire sale risk, maintaining the interpretation that run risk corresponds to failure at the

[3] Because we are taking the balance sheet as given, policy interpretations of comparative statics should be seem describing one channel, ignoring the impact of policy changes on the choice of balance sheet.

time of creditor withdrawals. We then study the decomposition of illiquidity risk into run risk and fire sale risk, in a piecewise linear model of balance sheet degradation, where withdrawals above some critical level imply a linear cost to the balance sheet interpreted as the cost associated with selling at fire sale prices. An important distinction is then that fire sale risk (unlike run risk) can arise even as solvency uncertainty becomes negligable.

The distinction between illiquidity risk and insolvency risk - within the same model but with incomplete information - arises in a number of contexts. An early reference is Postlewaite and Vives (1987). Most relevant for us is Rochet and Vives (2004), together with a follow up paper of Vives (2014). Rochet and Vives (2004) first applied global game methodology to study illiquidity risk for a bank with a stylized balance sheet like that considered here.[4] Our model can be seen as a stripped down version of their model, making even more simplifying assumptions to focus on key channels giving rise to the decomposition of credit risk. There are two substantive differences in our results/interpretations. First, their focus is on what we call fire sale risk, with our piecewise model of balance sheet degradation following theirs. As a result, illiquidity risk survives even without solvency risk. Second, they focus on when the bank fails. Second, by taking an ex ante perspective on illiquidity risk, we highlight the decreasing returns to increasing the liquidity ratio.

Other early papers using global game methods to study illiquidity risk are Morris and Shin (2004) and Goldstein and Pauzner (2005). The former gives a reduced form treatment of the ability of the bank to survive a run. The latter models partial repayment in a model of banks offering demand-deposits for the reasons highlighted in Diamond and Dybvig (1983). This paper follows Morris and Shin (2004) and Rochet and Vives (2004) in abstracting from partial repayment by assuming that all asset value is lost when a bank fails for illiquidity or solvency reasons. Schilling (2015) analyzes the implications of modelling partial repayment in models like this one. Eisenbach et al (2014) provide an analysis of illiquidity and insolvency risk with richer balance sheets but with a more reduced form (sunspot) treatment of resolution of the coordination problem.

[4] The focus of Rochet and Vives (2004) was on the role of lender of last resort policies. Vives (2014) focusses on comparative static and policy questions that are the focus of this paper.

2 Decomposition of Credit Risk

Consider the stylised balance sheet of a leveraged financial institution, called a "bank" for convenience. On the asset side, the bank holds two assets: a safe asset and a risky asset. We assume that the safe asset M ("bonds") is liquid and safe. The risky asset Y ("loans") is risky and cannot be sold (or repoed).[5] On the liability side, the bank has short term debt S and long term debt L. Writing E for the value of equity (Assets minus Liabilities), we thus have balance sheet

Assets	Liabilities
Cash M Risky Asset Y	Equity E Short Debt S Long Debt L

The interest rate on long and short debt and the return on cash and bonds are all normalized to be 0. At date 1, the gross return θ on the risky asset is uncertain. It is uniformly distributed on the interval $\left[\overline{\theta} - \frac{1}{2}\sigma, \overline{\theta} + \frac{1}{2}\sigma\right]$. Thus $\overline{\theta}$ is the mean return of the asset and σ is a measure of the variation. The return is realized at date 2. An important parameter in our analysis will be the liquidity ratio, specifying what proportion of creditors could be paid if all creditors ran: $\lambda = \frac{M}{S}$. A maintained assumption is that the liquidity ratio is less than 1, so that short term debt exceeds the assets than can be sold to pay it. Without this assumption, there will not be illiquidity risk.

2.1 Insolvency Risk

If nothing has happened before date 2, the equity of the bank is

$$M + \theta Y - S - L.$$

The bank is *solvent* at date 2 if this expression is positive, i.e., if

$$\theta \geq \theta^{**} = \frac{S + L - M}{Y}. \tag{1}$$

[5] For simplicity, we focus on two assets, one liquid and safe and one illiquid and risky. The analysis can be extended to allow independent variation in illiquidity and riskiness: see Morris and Shin (2010) on this issue.

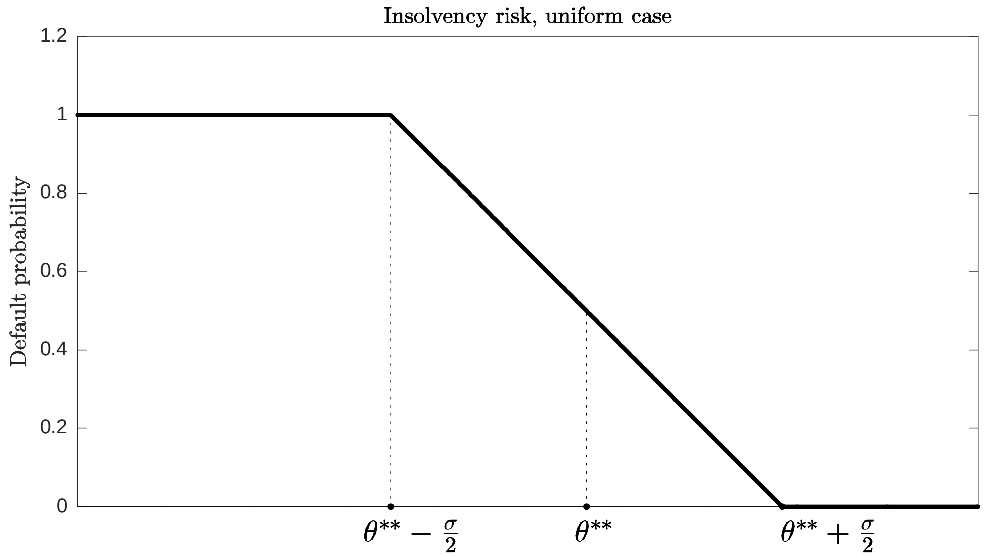

Figure 1: Insolvency Risk

We will refer to θ^{**} as *the solvency point.* We define *insolvency risk* at date 1 to be the probability that the bank fails under this scenario. Insolvency risk is then given by

$$\mathcal{S}\left(\overline{\theta}\right) = \Pr\left(\theta \leq \theta^{**}\right) = \begin{cases} 1\text{, if } \overline{\theta} \leq \theta^{**} - \frac{1}{2}\sigma \\ \frac{1}{2} + \frac{\overline{\theta} - \theta^{**}}{\sigma}\text{, if } \theta^{**} - \frac{1}{2}\sigma \leq \overline{\theta} \leq \theta^{**} + \frac{1}{2}\sigma \\ 0\text{, if } \theta^{**} + \frac{1}{2}\sigma \leq \theta_1 \end{cases}$$

Insolvency risk is plotted in figure 1.

2.2 Illiquidity Risk

Illiquidity risk is defined to be the risk that the bank fails due to a run at date 1 when it would have been solvent at date 2 without the run. Short term creditors have the option to rollover their debt at date 1, or to run (i.e., not rollover). If they choose to run, they have an outside option α with $0 < \alpha < 1$. Another maintained assumption will be that $\alpha < \lambda$. If this assumption failed, we will see that creditors would always want to run in equilibrium. If either the bank cannot cover short term withdrawals at date 1 or if the bank is insolvent at date 2, then debt holders receive no repayment. If both short term withdrawals are met at date 1 and the bank is solvent at date 2, then short term debt holders get a payment of 1.

There is thus a coordination problem for creditors. If proportion π of creditors run, then the bank will survive if

$$\pi S \leq M. \tag{2}$$

In calculating illiquidity risk, we initially assume[6] that - if the bank survives the run - there is no deterioration of the bank's balance sheet, even though assets are sold to meet withdrawals. Thus if the bank survives the run, it is possible to buy the assets back at par.

In characterizing illiquidity risk, assume for now that there is a critical θ_0^* below which all creditors run; and above which all creditors will rollover; and that at the critical θ_0^*, each creditor are indifferent between running and rolling over if he has a uniform belief over the proportion of others creditors running.[7] If π is uniformly distributed on the interval $[0, 1]$, the probability that condition 2 will be equal to the liquidity ratio $\lambda = \frac{M}{S}$. If the bank does survive, it may still be insolvent. So the expected return of short term creditors will be the probability that bank survives a run at date 1 times the probability that the bank is not insolvent at date 2. This gives an expected return of

$$\lambda \left(1 - \mathcal{S}\left(\overline{\theta}\right)\right)$$

Short run creditors will be indifferent between rolling over or running when this expression equals the outside option. This must occur when insolvency risk is between 0 and 1 and

$$\lambda \left(\frac{1}{2} + \frac{\theta^{**} - \overline{\theta}}{\sigma}\right) = \alpha \tag{3}$$

[6] The assumption is relaxed in section 3.

[7] The assumption will be endogenized in the next sub-section.

Write θ_0^* for the critical value of $\overline{\theta}$ where this holds. This implies

$$\theta_0^* = \theta^{**} + \sigma\left(\frac{\alpha}{\lambda} - \frac{1}{2}\right).[8]$$

Illiquidity risk is the probability that the bank fails due to a run when it would have survived in the event of a run. Illiquidity risk is therefore given by

$$\mathcal{I}\left(\overline{\theta}\right) = \begin{cases} 0, \text{ if } \overline{\theta} \leq \theta^{**} - \frac{1}{2}\sigma \\ \frac{1}{2} - \frac{1}{\sigma}\left(\theta^{**} - \overline{\theta}\right), \text{ if } \overline{\theta} \in \left[\theta^{**} - \frac{1}{2}\sigma, \sigma\left(\frac{\alpha S}{M} - \frac{1}{2}\right)\right] \\ 0, \text{ if } \overline{\theta} > \theta^{**} + \sigma\left(\frac{\alpha S}{M} - \frac{1}{2}\right) \end{cases}$$

Insolvency risk and illiquidity risk are plotted together in figure 2.

2.3 Aside: Global Game Foundation for Creditor Beliefs

It was assumed in the previous section that creditors had a uniform belief over the proportion of creditors running at the critical θ_0^*, and thus assigned probability λ to a run-induced bank failure. Suppose instead that creditors do not know $\overline{\theta}$ precisely and that their is variation in their expectations of $\overline{\theta}$. Assume that for sufficiently low $\overline{\theta}$ creditors would have a dominant strategy to run; and for sufficiently high $\overline{\theta}$ creditors would have a dominant strategy to rollover.[9] Say that a creditor has "rank belief" r if the proportion of creditors with a higher expectation of $\overline{\theta}$ is r. There is "common certainty of uniform rank beliefs" if each creditor always has a uniform belief about his rank independent of his expectation of $\overline{\theta}$. Morris, Shin and Yildiz (2016)

[8] Observe that under out maintained assumptions that $0 < \alpha < 1$ and $\alpha \leq \lambda \leq 1$, the expression $\frac{\alpha}{\lambda} - \frac{1}{2}$ is in the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and thus in insolvency risk is strictly between 0 and 1 at θ_0^*. As $\lambda \downarrow \alpha$, $\theta_0^* \rightarrow \theta^{**} + \frac{1}{2}\sigma$, and so runs always occur when insolvency risk is positive. As $\lambda \uparrow 1$, $\theta_0^* \rightarrow \theta^{**} + \sigma\left(\alpha - \frac{1}{2}\right)$ and thus runs happen the insolvency risk is greater than $1-\alpha$.

[9] The former condition is implied by payoffs in our game: for sufficiently low $\overline{\theta}$, creditors are certain that the bank is insolvent and will surely run. The latter condition is not implied by the payoffs, since a run can always occur for any $\overline{\theta}$. But the latter condition is easily justified by a natural perturbation of the model. Suppose that the bank had an extra risky asset on the balance sheet that could be used as collatoral to borrow an amount at date 1 that increases without bound in $\overline{\theta}$. An arbitrarily small about of this extra asset would ensure that there was a dominant strategy to rollover if $\overline{\theta}$ was sufficiently high.

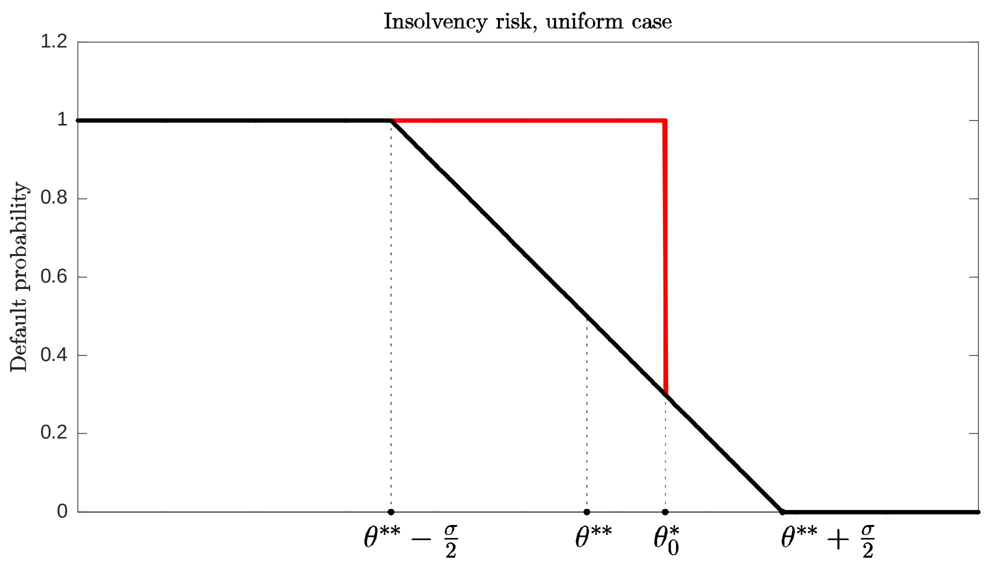

Figure 2: Illiquidity Risk

show if there is common certainty of uniform rank beliefs in games like that studied here, then there will be a unique strategy surviving iterated deletion of dominated strategies where a creditor withdraws if his signal is below his best response to uniform beliefs about his opponents' actions (i.e., θ_0^* in this example).[10] If agents' expectations of $\overline{\theta}$ are tightly distributed around the true $\overline{\theta}$, this argument rationalizes the assumption pinning down θ_0^* in the previous section.

Common certainty of rank beliefs is implied by assumptions made in the (symmetric) global games literature (Carlsson and Van Damme (1993) and Morris and Shin (1998)). In particular, suppose that each creditor observed a noisy signal of $\overline{\theta}$, $x_i = \overline{\theta} + \tau\varepsilon$, where the noise terms ε were distributed in the population according to $\varepsilon \sim f(\cdot)$. Assume that the prior distribution of $\overline{\theta}$ is given by density $g(\cdot)$. With this information, what rank beliefs will creditors hold? If $g(\cdot)$ is uniform, or if τ is small, the creditor will have uniform beliefs on π independent of x_i (Morris and Shin (2003)). Thus there will be common certainty of rank beliefs[11].

2.4 Ex Ante Credit Risk

It is possible to carry out comparative statics and policy analysis on the run points in the "interim" period 1. However, in analyzing comparative statics and policy, it is important to characterize illiquidity risk at an ex ante stage, where policy makers do not know exactly how much liquidity is needed prevent a run. Thus we consider a prior time 0, where $\overline{\theta}$ - the time 1 expectation of θ - is distributed uniformly on

$$\left[\theta_0 - \frac{1}{2}\xi, \theta_0 + \frac{1}{2}\xi\right].$$

We assume that ξ is large enough so that all $\overline{\theta} \in \left[\theta^{**} - \frac{1}{2}\sigma, \theta^{**} + \frac{1}{2}\sigma\right]$ are possible at date 0. Now ex ante illiquidity risk will be $\frac{1}{\xi}$ times the area of

[10] This is true under weaker conditions: it is enough that there is common certainty of approximately uniform rank beliefs conditional on there not being dominant actions. Morris, Shin and Yildiz (2016) show this for a class of games where payoffs are separable in others' actions and the state. This class does not include the "regime change" game discussed here, but the argument easily extends.

[11] More precisely, the weakening of common certainty of uniform rank beliefs described in footnote 10.

the illiquidity risk triangle in figure 2:

$$\frac{\sigma}{2\xi}\left(\frac{\alpha}{\lambda}\right)^2. \qquad \text{(ex ante liquidity risk)}$$

This expression gives the comparative statics discussed in the introduction: it is decreasing in the liquidity ratio, λ; increasing (and in fact linear) in solvency uncertainty, σ; and decreasing in the excess return offered to short term creditors over outside their outside options, $\frac{1}{\alpha}$. Also observe that the size of the derivative of liquidity risk with respect to its level, $\frac{\sigma}{\xi}\frac{\alpha^2}{\lambda^3}$, is decreasing in λ. The most interesting and robust channel driving this result is that the run point θ_0^* is decreasing in the liquidity ratio and the reduction in illiquidity risk associated with any reduction of θ_0^* is equal to 1 minus the insolvency risk. And the insolvency risk is decreasing in $\overline{\theta}$. In short, there is a small benefit to prevent runs when the probability of failure due to insolvency is high.

2.5 General Payoffs

In order to derive simple expressions and visualizations, it was convenient for us to focus on the case where θ had a uniform distribution. We will briefly note how the analysis extends to general distributions on θ. Beyond checking robustness and illustrating our results in a more general case, this extension will give a useful comparison with the analysis in the next section.

So now suppose that $\theta = \sigma\varepsilon$ where ε is drawn according to density $f(\cdot)$. Now the expression for the run point θ_0^*, analogous to (3), will be

$$\theta_0^* = \theta^{**} - \sigma F^{-1}\left(1 - \frac{\alpha}{\lambda}\right) \tag{4}$$

Insolvency and illiquidity risk are plotted figures 3 and 4 respectively.[12]

3 Balance Sheet Degradation

So far, we have maintained the assumption that if the bank survives the run, there is no long term damage to the balance sheet. However, illiquidity problems often lead to bank failure not because they cannot pay off their

[12] The figures are plotted for $M = 3$, $y = 1$, $S = 5$ and $L = 2$.

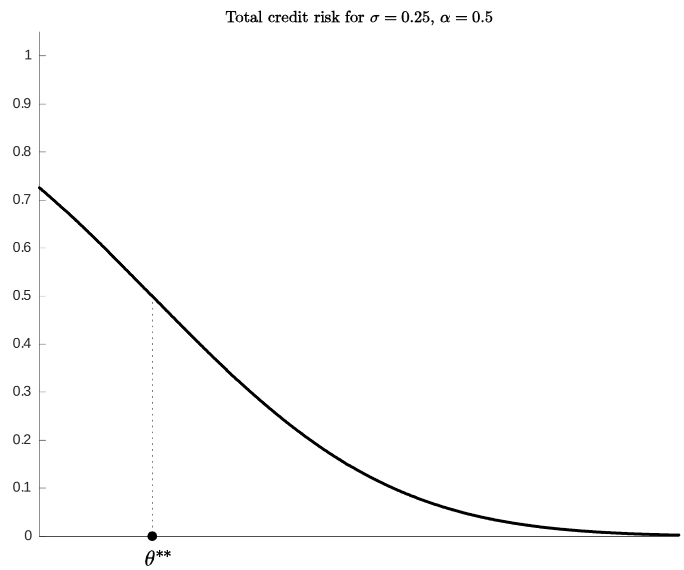

Figure 3: Normal Insolvency

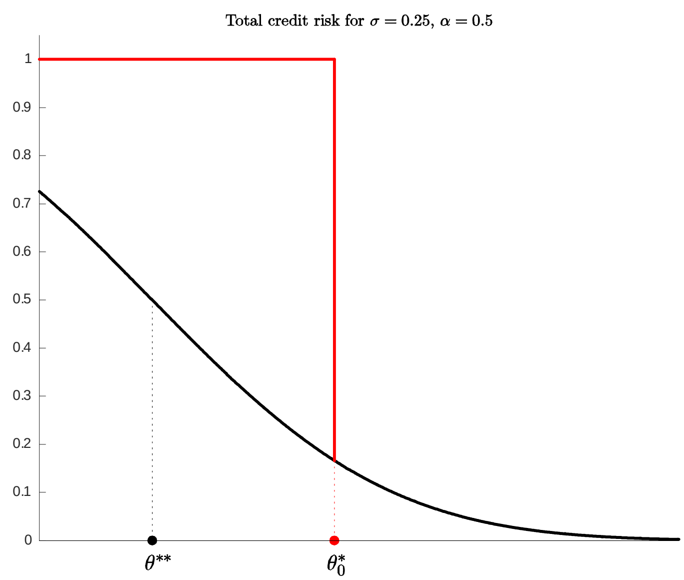

Figure 4: Normal Illiquidity

short term creditors, but rather because paying them off involves fire sales or expensive borrowing from other sources leading to eventual failure due to insolvency.

3.1 A Balance Sheet Degradation Interpretation of the Benchmark Model

Suppose now that the bank can always come up with enough cash to payoff short term creditors but the cost of doing so impairs the balance sheet. We can define an impairment function $\widetilde{\delta} : \mathbb{R}_+ \to \mathbb{R}_+$ where $\widetilde{\delta}(Z)$ is the cost to the balance sheet if proportion Z of creditors withdraw. Now the assumption in our benchmark model that the bank fails at the time of creditor withdrawals if those withdrawals exceed M can be replaced with the assumption that the impairment of balance sheet is ∞ if withdrawals exceed M. In this case, the bank does not fail at the time of withdrawals, but is doomed to failure later (a "zombie" bank) if survives to the next period. Now our benchmark model can be interpreted as one with only fire sale risk if

$$\widetilde{\delta}(Z) = \begin{cases} 0, \text{ if } Z \leq M \\ \infty, \text{ if } Z > M \end{cases} .$$

A less extreme assumption would be that $\widetilde{\delta}(Z)$ is increasing with $\widetilde{\delta}'(Z) \leq 1$ and convex when $Z < 1$. Now the interpretation: $\widetilde{\delta}(Z)$ is the price discount associated with Zth unit of assets sold, with assets with the smallest price discount sold first. A simple example of this impairment function would be a piecewise linear one with

$$\widetilde{\delta}(Z) = \begin{cases} 0, \text{ if } Z \leq M_0 \\ \delta(Z - M_0), \text{ if } M_0 \leq Z \leq M \\ \infty, \text{ if } M < Z \end{cases}$$

3.2 Another Risk Decomposition

Rather than re-interpreting the illiquidity risk in our benchmark model, we can also explicit model a natural decomposition of illiquidity risk into run risk and fire sale risk. We now suppose that the bank will fail in period 1 if withdrawals exceed M; the bank will survive with an unimpaired balance sheet if withdrawals are less than M_0; but if withdrawals Z are greater than

M_0 and less then M, the the bank will survive but the balance sheet will take a loss of $\delta\left(\pi S-M_0\right)$. With this interpretation, the piecewise linear impairment function is that studied by Rochet and Vives (2004) and Vives (2013). Now if we write π for the proportion of creditors who run, we have that if $M_0 \leq \pi S \leq M$, there be an adjusted solvency point of

$$\begin{aligned}
\theta_\delta^{**}(\pi) &= \frac{S+L+\delta\left(\pi S-M_0\right)-M}{Y} \\
&= \theta^{**}+\frac{\delta\left(\pi S-M_0\right)}{Y}
\end{aligned}$$

Now the expression for the run point θ_δ^*, analogous to (3) and ([**?**]), will be

$$\frac{M_0}{S} F\left(\frac{\theta^{**}-\theta_\delta^*}{\sigma}\right)+\int\limits_{\pi=\frac{M_0}{S}}^{\frac{M}{S}} F\left(\frac{\theta^{**}-\theta_\delta^*+\frac{\delta}{Y}\left(\pi S-M_0\right)}{\sigma}\right)=\alpha$$

We cannot solve this in closed form. But we can compute θ_δ^* and identify fire state risk with one minus the insolvency risk when $\theta \in\left[\theta_0^*, \theta_\delta^*\right]$. Fire sale risk is added into figure 5. We can now perform comparative statics on solvency uncertainty. Increasing solvency uncertainty give us figure 6. Decreasing solvency uncertainty gives figure 7. Decreasing solvency uncertainty further gives us figure 8. .

The figures illustrate a striking property: as $\sigma \rightarrow 0$, "run risk" disappears but fire sale risk does not disappear. In fact, we can give a closed form expression for the fire sale point when $\sigma \rightarrow 0$. If $\bar{\theta}$ is just above θ^{**}, balance sheet degradation will imply eventual failure even when solvency uncertainty is small. Suppose that $M_0 \leq \alpha S \leq M$. In this case, if the fire sale point were θ_δ^*, there will not be a run as $\sigma \rightarrow 0$, only if π satisfies

$$\theta^{**}+\frac{\delta\left(\pi S-M_0\right)}{Y} \geq \theta_\delta^*$$

Re-arranging gives the critical proportion of creditors at which a run will occur:

$$\pi_\delta^*=\frac{1}{S}\left(\frac{\left(\theta_\delta^*-\theta^{**}\right) Y}{\delta}+M_0\right)$$

Now using the uniform belief about the proportion of creditors running gives

$$\frac{1}{S}\left(\frac{\left(\theta_\delta^*-\theta^{**}\right) Y}{\delta}+M_0\right)=\alpha$$

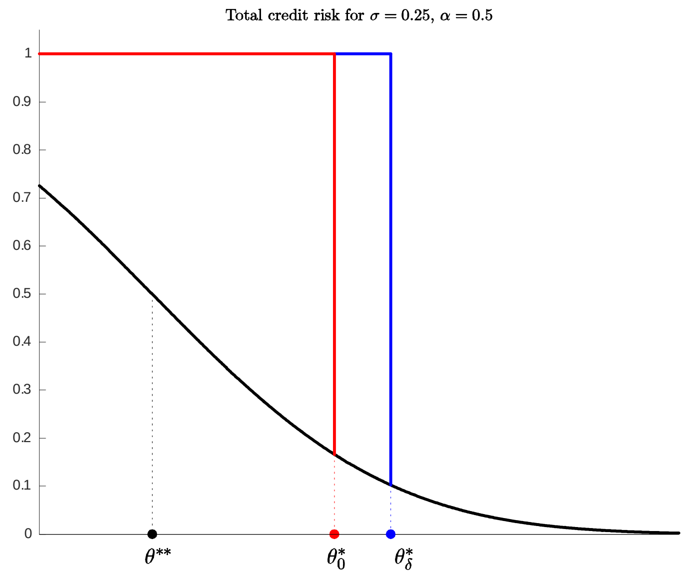

Figure 5: Normal Fire Sale

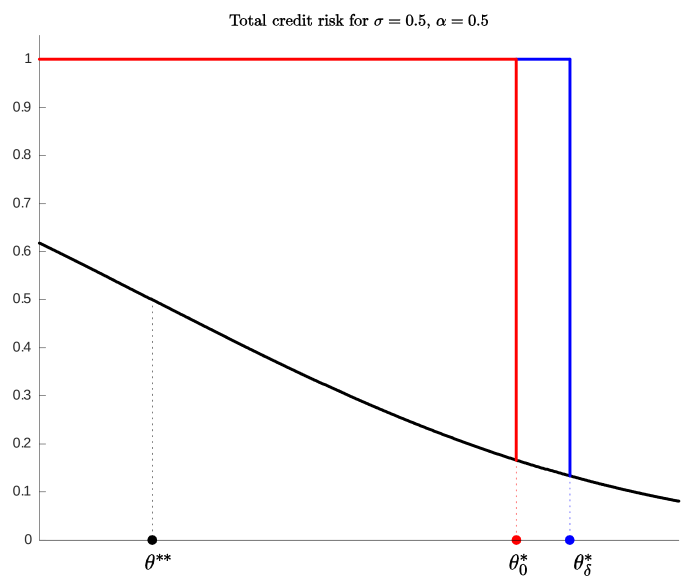

Figure 6: High Solvency Uncertainty

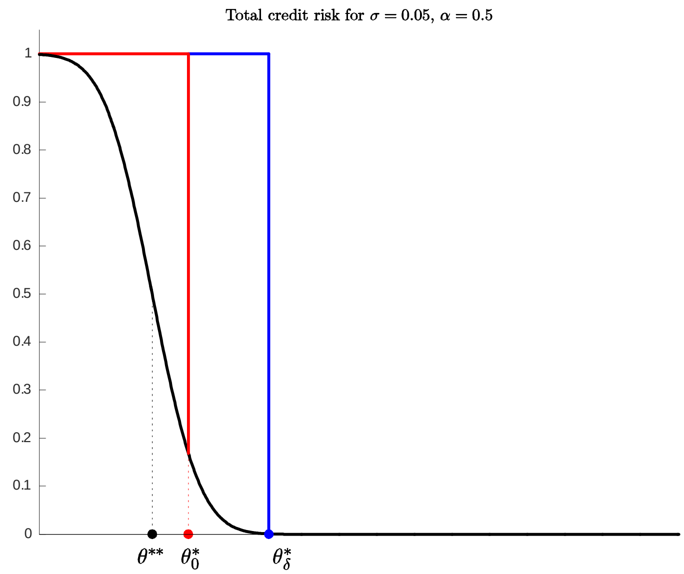

Figure 7: Low Solvency Uncertainty

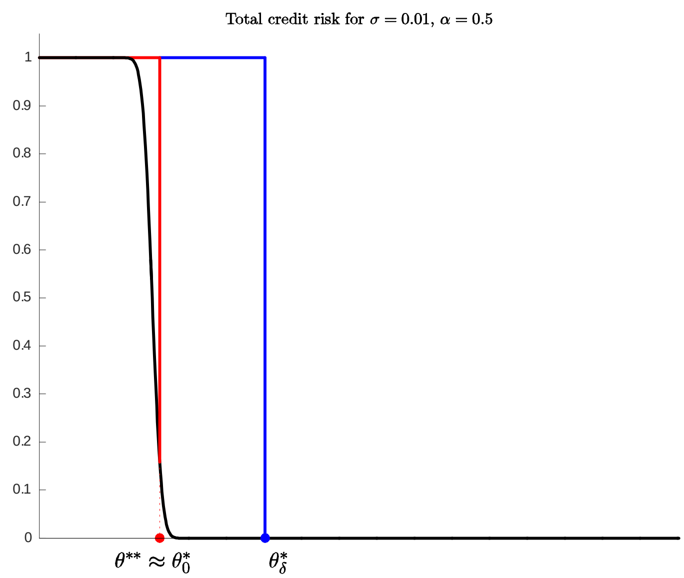

Figure 8: Very Low Solvency Uncertainty

But this in turn gives an expression for the fire sale point as $\sigma \to 0$:

$$\theta_\delta^* \to \theta^{**} + \delta \left(\frac{\alpha S - M_0}{Y} \right)$$

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