

Currency Wars, Trade Wars and Global Demand

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Abstract

I present a tractable model of a global economy with downward nominal wage rigidity in which countries attempt to boost their employment and welfare by depreciating their currencies and making their goods more competitive—a "currency war"—or by imposing a tariff on imports—a "trade war." A currency war can be waged with a range of policy instruments—by lowering the nominal interest rate, by raising the inflation target or by capital controls. Trade wars depress global demand and employment and may lead to large welfare losses (amounting to more than 10 percent of potential GDP under our benchmark calibration). The welfare effects of currency wars depend on the policy instruments but are nonnegative when all countries use the same instruments. The uncoordinated use of capital controls may lead to symmetry breaking, with a fraction of countries competitively devaluing their currency and lending their surpluses to deficit countries at a low interest rate. Currency wars may lead to a large welfare transfer from the countries that do not use capital controls to those that do.

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1 Introduction

Countries have regularly accused each other of being aggressors in a currency war since the global financial crisis. Guido Mantega, Brazil's finance minister, in 2010 accused the US of launching "currency wars" through quantitative easing and a lower dollar. "We're in the midst of an international currency war, a general weakening of currency. This threatens us because it takes away our competitiveness."¹ At the time Brazil itself was trying to hold its currency down by accumulating reserves and by imposing a tax on capital inflows. Many countries, including advanced economies such as Switzerland, have depreciated or resisted the appreciation of their currency by resorting to foreign exchange interventions. The phrase "currency war" was again used when the Japanese yen depreciated in 2013 after the Bank of Japan increased its inflation target (and more recently when it reduced the interest rate to a negative level). Bergsten and Gagnon (2012) propose that the US undertake countervailing currency intervention against countries that manipulate their currencies, or tax the earnings on the dollar assets of these countries. The election of Donald Trump added to these concerns that of a tariff war initiated by the US.

While G20 countries have regularly renewed their pledge to avoid depreciating their currencies to gain a competitive trading advantage, they have also implemented stimulative policies that often led to depreciation. Bernanke (2015) argues that this situation should not raise concerns about currency wars as long as the depreciations are the by-product, rather than the main objective, of monetary stimulus (see also Blanchard (2016)). Mishra and Rajan (2016) find the international spillovers from monetary and exchange rate policies less benign and advocate enhanced international coordination to limit the effects of these spillovers.

The concepts of currency war and trade war are old ones but we do not have many models to analyze these wars, separately or as concurrent phenomena (more on this in the discussion of the literature below). In this paper I present a simple model in which an open economy can increase its employment and welfare by depreciating its currency and making its goods more

¹Reported by Martin Wolf in "Currencies Clash in New Age of Beggar-thy-Neighbor," Financial Times September 28, 2010.

competitive in exports markets. I consider a symmetric world with many identical countries, each one producing its own good like in Galí and Monacelli (2005). There is downward nominal stickiness in wages like in Schmitt-Grohé and Uribe (2016). I characterize the Nash equilibria in exchange rate and trade policies and explore the case for international coordination. The main qualities that I look for in the model are tractability and analytical transparency but the model can be used to quantify the size of the effects, and in particular the welfare cost of currency and trade wars.

The main results crucially depend on whether global demand is sufficient to ensure full employment at the global level. If there is full employment at the global there is no need for international monetary cooperation but it is beneficial to agree on not using tariffs. Exchange rate policy is used to achieve full employment and tariffs are used to manipulate the terms of trade. Individual countries are not tempted to engage in a currency war since there is full employment. The temptation, rather, is for each country to manipulate the terms of trade in its favor by appreciating its currency while maintaining full employment with a tariff on imports. The outcome, in general equilibrium, is that international trade is inefficiently low and consumption is distorted towards the home good, like in the textbook model of tariff war. Under plausible calibrations the welfare cost of these trade wars is equivalent to a permanent decrease in consumption of one or two percent (significant but not overwhelming).

The results are different when global demand is insufficient. This scenario is obtained by increasing the discount factor of the representative consumer up to a point where the global economy falls in a liquidity trap. The nominal interest rate is at the zero bound and there is unemployment in all countries. Each country is tempted to boost its own employment by increasing its share in global demand but the collective implications of such beggar-thy-neighbor policies crucially depend on which policy instrument is used. There is no benefit from coordinating conventional monetary policy. There is also no benefit from coordinating unconventional monetary policy that manipulates the inflation target. In fact, the Nash equilibrium in which each country sets its inflation target competitively leads to full employment as it relaxes the zero bound constraint on the real interest rate.

The case for international coordination is the strongest when it comes to prevent a tariff war in a global liquidity trap. In a liquidity trap countries use tariffs in order to reach employment objectives. Each country finds it optimal to impose a tariff in order to switch domestic demand away from

imports and towards the home good to increase domestic employment. This makes sense in partial equilibrium but decreases each country's contribution to global demand. Global demand and employment are lower in the Nash equilibrium with tariffs. The welfare impact of a tariff war can be substantial: under my benchmark calibration the unemployment rate is increased from 10 percent to about 18 percent.

I also look at the case where countries can depreciate their currencies by restricting capital inflows and accumulating reserves (still in the case of a global liquidity trap), a situation that has been called a "capital war." I find that under my benchmark calibration a capital war leads to endogenous symmetry-breaking. A fraction of countries accumulate foreign assets to achieve a trade surplus and full employment, whereas the other countries accept a trade deficit and less than full employment. The welfare of deficit countries, however, is the same as that of surplus countries because they can borrow at a very low cost while the surplus countries receive a very low return on their external assets. Furthermore, global welfare is slightly increased by the capital war.

Literature. There is a long line of literature on international monetary coordination—see e.g. Engel (2016) for a review. The case for international monetary cooperation in New Open Macro models was studied by Obstfeld and Rogoff (2002), Benigno and Benigno (2006), Canzoneri, Cumby and Diba (2005) among others. This line of literature has concluded that the welfare cost of domestically-oriented rules is small.

A more recent group of papers has explored the international spillovers associated with monetary policy when low natural rates of interest lead to insufficient global demand and liquidity traps: Eggertsson et al. (2016), Caballero, Farhi and Gourinchas (2015), Fujiwara et al. (2013), Devereux and Yetman (2014), Cook and Devereux (2013), and Acharya and Bengui (2016). This paper shares some themes with that literature, in particular the international contagion in the conditions leading to a liquidity trap. Eggertsson et al. (2016) and Caballero, Farhi and Gourinchas (2015) study the international transmission of liquidity traps using a model that shares several features with this paper, in particular the downward nominal stickiness à la Schmitt-Grohé and Uribe (2016).

This paper is related to the recent literature that looks at the macroeconomic impact of trade policy. Barbiero et al. (2017) study the macroeconomic consequences of a border adjustment tax in the context of a dynamic general

equilibrium model with nominal stickiness and a monetary policy conducted according to a conventional Taylor rule. Lindé and Pescatori (2017) study the robustness of the Lerner symmetry result in an open economy New Keynesian model with price rigidities and find that the macroeconomic costs of a trade war can be substantial. Erceg, Prestipino and Raffo (2017) study the short-run macroeconomic effects of trade policies a dynamic New Keynesian open-economy framework.

In our model the social planner uses capital controls to affect the exchange rate, a form of intervention that can be interpreted as foreign exchange interventions. Fanelli and Straub (2016) present a model in which countries can use foreign exchange interventions to affect their terms of trade. A two-period version of their model features a Nash equilibrium between advanced economies and emerging market economies where the latter accumulate reserves to depreciate their real exchange rate. This equilibrium is inefficient and there is scope for international coordination to reduce reserve accumulation. Amador et al. (2017) study the use of foreign exchange interventions at the zero lower bound.

The paper is related to and Korinek (2016). That paper presents a unified framework for analyzing whether international spillovers call for international policy coordination. Korinek gives a set of conditions under which the spillovers are efficient and coordination is uncalled for. The model in this paper does not satisfy these conditions—in particular the fact that countries do not have monopoly power.

2 Assumptions

The model represents a world composed of a continuum of atomistic countries indexed by $j \in (0, 1)$ over a finite number of periods $t = 1, 2, \dots, \theta$. The goods structure is similar to Galí and Monacelli (2005). Each country produces its own good and has its own currency. The nominal wage is rigid downwards as in Schmitt-Grohé and Uribe (2016, 2017). There is no uncertainty.

Each country is populated by a mass of identical consumers. I first describe the preferences of the representative consumer (omitting the country index to alleviate notations).

Preferences. In all periods except the final one, the utility of the rep-

representative consumer is defined recursively by,

$$U_t = u(C_t) + e^{-\rho t} U_{t+1}. \quad (1)$$

Time variation in the logarithmic discount rate ρ_t will be used to affect domestic demand. The utility function has a constant relative risk aversion

$$u(C) = C^{1-1/\sigma} / (1 - 1/\sigma).$$

The consumer consumes the good that is produced domestically (the home good) as well as a basket of foreign goods. In all periods except the final one the consumer cares about the Cobb-Douglas index,

$$C = \left(\frac{C_H}{\alpha_H} \right)^{\alpha_H} \left(\frac{C_F}{\alpha_F} \right)^{\alpha_F}, \quad (2)$$

(with $\alpha_H + \alpha_F = 1$) where C_H is the consumption of home good, and C_F is the consumption of foreign good.

Utility in the final period varies linearly with the consumption of the two goods,

$$U_\theta = C_{H\theta} + C_{F\theta}.$$

This specification implies that the final period terms of trade are equal to 1 independently of the country's net foreign assets, which makes the model tractable. We present in section [.] a more general version of the model with infinite time and show there that the quantitative implications are very close to those of the baseline model.

The consumption of foreign good is a CES index of the goods produced in all the countries,

$$C_F = \left[\int_0^1 C_k^{(\gamma-1)/\gamma} dk \right]^{\gamma/(\gamma-1)}.$$

Imports are defined by the same index for all countries. The composite good defined by this index will be called the "global good" in the following. The elasticity of substitution between foreign goods is assumed to be larger than one, $\gamma > 1$.

Production and labor market. The home good is produced with a linear production function that transforms one unit of labor into one unit of

good, $Y = L$. The representative consumer is endowed with a fixed quantity of labor \bar{L} and the quantity of employed labor satisfies

$$L \leq \bar{L}. \quad (3)$$

There is full employment if this constraint is satisfied as an equality. It is assumed that there is full employment in the final period, $L_\theta = \bar{L}$, but there could be unemployment in earlier periods. We normalize \bar{L} to 1.

The period- t nominal wage is denoted by W_t . Like in Schmitt-Grohé and Uribe (2016) or Eggertsson et al. (2016), downward nominal stickiness in the wage is captured by the constraint,

$$W_t \geq \underline{\Pi} W_{t-1}. \quad (4)$$

where $\underline{\Pi}$ is an exogenous parameter that is equal to or lower than 1. In any period t the economy can be in two regimes: full employment ($L_t = \bar{L}$), or less than full employment, in which case the nominal wage is at its lower bound ($L_t < \bar{L}$ and $W_t = \gamma W_{t-1}$). The constraints on the labor market can be summarized by (3), (4) and

$$(\bar{L} - L_t)(W_t - \underline{\Pi} W_{t-1}) = 0. \quad (5)$$

This leads to a L-shaped Phillips curve where the nominal wage can be set independently of employment once there is full employment. The gross inflation rate in the nominal wage (or home good price) is denoted by

$$\Pi_t \equiv \frac{W_t}{W_{t-1}}.$$

Budget constraints. Consumers trade one-period bonds denominated in the global good. In any period $t < T$ the budget constraint of the representative consumer is

$$P_t \frac{B_{t+1}}{R_t T_t^b} + W_t C_{Ht} + T_t^m P_t C_{Ft} = W_t L_t + Z_t + P_t B_t, \quad (6)$$

where P_t denotes the offshore domestic-currency price of the global good, T_t^m is a gross tax on imports, T_t^b is a gross tax on foreign borrowing, B_t is the quantity of real bonds held by the representative consumer at the beginning of period t , R_t is the gross real interest rate in terms of the global good and

Z_t is the lump-sum rebate of the proceeds of the taxes. I have used the fact that the price of the home good is equal to the wage because of the linearity in the production function.

In the final period the budget constraint is given by,

$$W_\theta C_{H\theta} + P_\theta C_{F\theta} = W_\theta \bar{L} + P_\theta B_\theta. \quad (7)$$

There is full employment and no trade tax in the final period. As a result the terms of trade are equal to 1 ($W_\theta = P_\theta$), and welfare is given by,

$$U_\theta = \bar{L} + B_\theta. \quad (8)$$

The period- t demand for home labor is,

$$L_t = C_{Ht} + \left[T_t^x \frac{W_t}{P_t} \right]^{-\gamma} C_{Ft}^W, \quad (9)$$

where $C_F^W = \int C_{Fk} dk$ denotes global gross imports and T^x is the gross tax on exports. The first term on the right-hand side of (9) is the labor used to serve the home demand for the home good and the second term is the labor used to produce exports.

It will be convenient to define three terms of trade,

$$S_t \equiv \frac{W_t}{P_t}, \quad S_t^m \equiv \frac{S_t}{T_t^m} \quad \text{and} \quad S_t^x \equiv T_t^x S_t, \quad (10)$$

where S_t denotes the undistorted terms of trade, and S_t^m and S_t^x are the tax-distorted terms of trade that apply to imports and exports respectively. Given the Cobb-Douglas specification (2) the home demand for the home good and for imports are respectively given by,

$$C_{Ht} = \alpha_H (S_t^m)^{-\alpha_F} C_t, \quad (11)$$

$$C_{Ft} = \alpha_F (S_t^m)^{\alpha_H} C_t. \quad (12)$$

The demand for home labor (9) can thus be re-written,

$$L_t = \alpha_H (S_t^m)^{-\alpha_F} C_t + (S_t^x)^{-\gamma} C_{Ft}^W. \quad (13)$$

The demand for home labor increases with home consumption and global consumptions but is reduced by a loss in competitiveness (an increase in S^m or S^x).

Finally, using $Z_t = (T_t^m - 1)P_t C_{Ft} + (T_t^x - 1)W_t(L_t - C_{Ht}) - (1 - 1/T_t^b)P_t B_{t+1}/R_t$ and equations (9), (10), and (12) to substitute out Z_t , L_t , C_{Ht} , C_{Ft} , W_t and P_t from the representative consumer's budget constraint (6) gives the balance of payments equation

$$\frac{B_{t+1}}{R_t} = B_t + X_t, \quad (14)$$

where net exports in terms of global good are given by

$$X_t = (S_t^x)^{1-\gamma} C_{Ft}^W - \alpha_F (S_t^m)^{\alpha_H} C_t. \quad (15)$$

Net exports are a function of domestic and global consumption, and of the terms of trade that are relevant for exports and imports. Note that the value of net exports in terms of global good decreases if the country loses competitiveness in export markets (an increase in S^x) because $\gamma > 1$.

3 Policies

We now review the policy instruments that are available to the domestic social planner. There are three policy areas: monetary policy, trade policy, and capital account policy (or capital controls). This section shows that the policy instruments are well-defined in the sense that they determine one unique allocation. The second subsection establishes equivalence results between capital account policy and trade policy, the third subsection characterizes the impact of the policy instruments on the allocations, and the last subsection looks at optimal policy.

3.1 Policy instruments

The index j is introduced to distinguish country variables from global variables. The main instrument of monetary policy is the nominal interest rate, denoted by i_{jt} . The nominal interest rate is defined as the logarithmic nominal return on onshore domestic currency bonds, i.e., $\exp(-i_{jt})$ is the period- t price of a claim to one unit of domestic currency in the next period. The nominal interest rate can be set freely subject to the zero lower bound (ZLB) constraint $i_{jt} \geq 0$. Using logarithmic returns alleviates the algebra because several equilibrium relationships in the model are linear in logarithm. We generally use a lower-case letter to denote the logarithm of a variable.

We denote log inflation in the price of the home good by $\pi_{jt} = \log(W_t/W_{t-1})$. By assumption the inflation rate is not under the control of the monetary authorities and is equal to $\underline{\pi} \equiv \log \underline{\Pi}$ when there is unemployment. We assume that when there is full employment the inflation rate is set according to an inflation targeting rule.² The inflation rate is equal to a target π_j^* whenever there is full employment

$$\pi_{it} = \pi_j^* \text{ if } L_{jt} = \bar{L}, \quad (16)$$

$$= \underline{\pi} \text{ if } L_{jt} < \bar{L}. \quad (17)$$

The instruments of trade policy are the taxes on imports and on exports. Like for returns we use logarithmic taxes,

$$\tau_{jt}^m = \log T_{jt}^m, \quad \tau_{jt}^x = \log T_{jt}^x.$$

The instrument of capital account policy is the tax on capital inflows (or subsidy on outflows) $\tau_{jt}^b = \log T_{jt}^b$. As we show at the end of this section, the instrument of capital account policy could instead be specified as foreign exchange interventions when the capital account is closed (Jeanne, 2013).

To summarize, the policy of country j is characterized by a path $(i_{jt}, \tau_{jt}^m, \tau_{jt}^x, \tau_{jt}^b)_{t=1, \dots, \theta-1}$. I now show that conditional on the inflation targeting rule (16) a given policy determines one unique allocation. For this I rely on two equilibrium conditions.

First, arbitrage between real and nominal bonds by residents implies

$$\begin{aligned} \exp(i_t) &= R_t T_t^b \frac{P_{t+1}}{P_t}, \\ &= R_t T_t^b \Pi_{t+1} \frac{S_t}{S_{t+1}}. \end{aligned}$$

Taking the log of this expression, iterating forward and using the fact that the final terms of trade are equal to one ($S_\theta = 1$) then gives an expression for the period- t terms of trade

$$s_{jt} = \sum_{t'=t}^{\theta-1} (i_{jt'} - \pi_{jt'+1} - r_{t'} - \tau_{jt'}^b). \quad (18)$$

²Appendix A shows how the inflation rate can be set to the target using money supply. We abstract from money supply in the baseline model and assume that the monetary authorities can implement the target conditional on full employment.

The log terms of trade are equal to the cumulative difference between the home good own rate of interest and the global good own rate of interest. The terms of trade of country j can be reduced (its currency depreciated in real terms) by a decrease in the current or future nominal interest rate or an increase in the current or future tax on capital flows. Thus, the interest rate and capital controls can be viewed as alternative instruments of exchange rate policy.

Second, the Euler equation for the representative consumer implies

$$c_{jt} = -\sigma \left[\alpha_H \sum_{t'=t}^{\theta-1} (i_{jt'} - \pi_{jt'+1} - \rho_{jt'}) + \alpha_F \sum_{t'=t}^{\theta-1} (r_{t'} + \tau_{jt'}^b - \rho_{jt'}) + \alpha_F \tau_{jt}^m \right], \quad (19)$$

(this equation is derived in Appendix A). Log consumption is equal to minus the intertemporal elasticity of substitution times the cumulative average of the home and foreign goods own rates of interest weighted by the share of each good in the utility function, plus a term that decreases with the current-period tax on imports. This term reflects the fact that the tax on imports is a tax on consumption and thus affect the consumer's intertemporal consumption-saving choice.

Given the policy $(i_{jt}, \tau_{jt}^m, \tau_{jt}^x, \tau_{jt}^b)_{t=1, \dots, \theta-1}$ the equilibrium can be constructed by iterating backwards using (18) and (19). In period θ there is full employment and the inflation rate is equal to the target. In period $\theta - 1$, because $\pi_{j\theta} = \pi_j^*$ equations (18) and (19) give $s_{j\theta-1}$ and $c_{j\theta-1}$ conditional on policy $i_{j\theta-1}$ and $\tau_{j\theta-1}^b$. The demand for labor $L_{j\theta-1}$ is derived from (13) conditional on the trade taxes $\tau_{j\theta-1}^m$ and $\tau_{j\theta-1}^x$. The inflation rate $\pi_{j\theta-1}$ then results from the rule (16). The same iteration can be implemented going backward to $t = 1$, the key point of each iteration being to determine whether there is full employment or not.

Proposition 1 *Conditional on the inflation targeting rule (16) any policy $(i_{jt}, \tau_{jt}^m, \tau_{jt}^x, \tau_{jt}^b)_{t=1, \dots, \theta-1}$ determines at most one allocation $(C_{Hjt}, C_{Fjt}, C_{jt}, L_{jt}, \Pi_{jt})_{t=1, \dots, \theta}$ satisfying the equilibrium conditions.*

Proof. See discussion above. ■

Although the allocation is unique, it may not correspond to an equilibrium because the constraint $L_{jt} \leq \bar{L}$ is not necessarily satisfied. In the following we consider *admissible* policies that satisfy this constraint.

The instrument of capital account policy could instead be specified as foreign exchange interventions (Jeanne, 2013). To see this, assume that the capital account is closed, i.e. the that government monopolizes financial transactions with the rest of the world and holds all the country's foreign assets B_t (which could be interpreted as foreign exchange reserves).³ Then a policy consists in a path $(i_{jt}, \tau_{jt}^m, \tau_{jt}^x, B_{jt+1})_{t=1, \dots, \theta-1}$ and still determines one unique allocation, as stated in the following proposition.

Proposition 2 *Assume that the capital account is closed and the government determines the level of net foreign assets (reserves) B . Conditional on the inflation targeting rule (16) any policy $(i_{jt}, \tau_{jt}^m, \tau_{jt}^x, B_{jt+1})_{t=1, \dots, \theta-1}$ determines at most one allocation $(C_{Hjt}, C_{Fjt}, C_{jt}, L_{jt}, \Pi_{jt})_{t=1, \dots, \theta}$ satisfying the equilibrium conditions.*

Proof. See Appendix C. ■

Taxing capital flows and reserves interventions are two different ways of achieving the same allocations. For example, the impact of increasing the tax on capital flows in a given period t can be reproduced, with a closed capital account, by increasing the foreign exchange reserves B_{jt+1} .

3.2 Equivalence between exchange rate policy and trade policy

A long-standing question in the macroeconomic and trade literature is that of the conditions under which exchange rate manipulation can replicate the impact of tariffs. The relationship between trade policy and exchange rate policy is clarified in the following proposition.

Proposition 3 *Any allocation $(C_{Hjt}, C_{Fjt}, L_{jt}, \Pi_{jt})_{t=1, \dots, \theta}$ achieved by policy $(i_{jt}, \tau_{jt}^m, \tau_{jt}^x, \tau_{jt}^b)_{t=1, \dots, \theta-1}$ can also be achieved without export tax by policy $(i_{jt}, \tau_{jt}^m + \tau_{jt}^x, 0, \tau_{jt}^b + \tau_{jt+1}^x - \tau_{jt}^x)_{t=1, \dots, \theta-1}$.*

Proof. See Appendix C. ■

³The assumption that there are no private capital flows is extreme but the insights remain true if frictions prevent economic agents from arbitraging the wedge between onshore and offshore interest rates.

In words, the allocation is unchanged if one shifts the tax on exports to imports and at the same time increase the tax on capital inflows by the one-period-ahead change in the tax on exports.⁴ For the gross exports to be left unchanged, the decrease in the export tax must be perfectly offset by an increase in the terms of trade (a real appreciation). The real appreciation must be offset by an equivalent increase in the tax on imports to keep imports the same. The real appreciation results from a decrease in the tax on capital inflows that is cumulatively of the same size as the tax on exports but distributed over time so as to offset the intertemporal impact of the tax on imports.

The proposition allows us to identify the conditions under which trade policy and exchange rate policy are equivalent (Meade, 1955). Consider the trade policies that subsidize exports at the same rate as they tax imports, that is

$$\tau_{jt}^m + \tau_{jt}^x = 0, \quad (20)$$

for all times t . This condition implies that $s_{jt}^m = s_{jt}^x$, i.e., the trade taxes induce the same distortion for imports and exports. Then Proposition 3 implies that the real allocations achieved by trade policy can be replicated, without any trade tax, by a tax on capital inflows.⁵ That is, there is an equivalence between exchange rate policy and trade policy provided that (i) trade policy introduces the same terms of trade distortion in domestic and foreign markets; and (ii) the instrument of exchange rate policy is the tax on capital flows rather than the interest rate.

Another implication of Proposition 3 is that one of the policy instruments is redundant. If the country does not peg the exchange rate it does not need to have taxes on both imports and exports. Hereafter I will assume that the social planner does not tax exports

$$\tau_{jt}^x = 0.$$

From now on policy will be defined for each country j as a path of three instruments $(i_{jt}, \tau_{jt}^m, \tau_{jt}^b)_{t=1, \dots, \theta-1}$.

⁴The fact that a tax on imports has the same impact as a tax on exports is known as Lerner's symmetry theorem in the trade literature (Lerner, 1936). Costinot and Werning (2017) provide a number of generalizations and qualifications of the Lerner symmetry theorem.

⁵The equivalence between taxes on trade and taxes on capital flows is studied in Costinot, Lorenzoni and Werning (2014).

3.3 Comparative statics

This section explains how policy instruments impact allocations. We illustrate the properties of the model based on numerical values but the properties are general except when otherwise noted.

We assume the following parameter values in quantitative illustrations of the model. The elasticity of intertemporal substitution of consumption, σ , is set to 0.5, which corresponds to a risk aversion of 2, a standard value in the literature. The elasticity of substitution between foreign goods, γ , is set to 3, which is consistent with the recent estimates of Feenstra et al. (2018). Note in particular that the "microelasticity" between the differentiated imported goods is substantially larger than the "macroelasticity" between the home good and imports (which is 1 because of the Cobb-Douglas specification). Finally, we assume $\alpha_H = 0.6$, i.e., home goods amount to 60 percent of total consumption.

Table 1 shows how the terms of trade, consumption, employment, net exports and welfare are contemporaneously affected by a change in any given policy instrument. The table reports the semi-elasticity of the terms of trade S_j , consumption C_j and employment L_j with respect to the row variable.⁶ For the net exports X_j and the country's welfare U_j the semi-elasticities are normalized by the level of consumption. For these two variables the semi-elasticities with respect to policy instrument $n = i, \tau^m, \tau^b$ are given by $\frac{1}{C_j} \frac{\partial B_j}{\partial n_j}$ and $\frac{1}{C_j} \frac{\partial U_j}{\partial n_j}$. The elasticities are computed in a symmetric undistorted allocation in which $\tau_{jt}^m = \tau_{jt}^b = 0$ and assuming less than full employment. For simplicity it is further assumed that $r_t = \rho_t$ in all periods.

Several observations are in order. First, the elasticities of employment and welfare with respect to all policy instruments have the same signs. This means that any policy that raise employment also raises welfare independently of the impact that it has on other variables. One should not infer from this result that maximizing welfare is always equivalent to reaching full employment because the elasticities reported in Table 1 apply only around a symmetric undistorted allocation. We will indeed see that under some circumstances welfare-maximizing social planners do not seek full employment.

Second, the import tariff raises employment and welfare. This is not true in general and depends on how the elasticity of substitution between the two goods compares with the elasticity of intertemporal substitution of

⁶The derivation of these elasticities can be found in Appendix B.

consumption. The tariff has an ambiguous effect on employment because it reduces total consumption at the same time as it shifts consumption towards the home good. The tariff raises employment if the second effect dominates, that is if the elasticity of intertemporal substitution of consumption is smaller than the elasticity of substitution between the two goods, $\sigma < 1$. This condition is satisfied by our benchmark calibration.

Third, a tariff on imports and a tax on capital inflows have similar effects except that the tax on capital inflows reduces the terms of trade whereas the tariff on imports does not. This implies that the tax on capital inflows has a larger impact than the tariff. A one-percent decrease in the nominal interest and a one-percent increase in the capital inflow tax have very similar impact, both in signs and magnitude. The impact of a tariff on imports is significantly smaller because, as noted above, it does not affect exports and the elasticity of substitution between home and foreign goods is significantly smaller in the home market than in foreign markets.

The elasticities reported in Table 1 assume less than full employment. When there is full employment the policy changes considered in Table 1 are not necessarily feasible. For example, it is not possible to the tax on imports because this would increase employment above \bar{L} . The increase in the tax on imports must be associated with other policy changes that ensure that employment stays below \bar{L} , for example an increase in the nominal interest rate or an increase in inflation above the target that appreciates the currency in real terms.

Table 1. Semi-elasticities of macroeconomic variables and welfare with respect to policy instruments under benchmark calibration

	i_{jt}	τ_{jt}^m	τ_{jt}^b
S_{jt}	+1.0	0.0	-1.0
C_{jt}	-0.3	-0.2	-0.2
L_{jt}	-1.6	+0.1	+1.3
X_{jt}	-0.9	+0.3	+1.1
U_{jt}	-0.9	+0.1	+0.9

3.4 Optimal policy

This section considers the policies $(i_{jt}, \tau_{jt}^m, \tau_{jt}^b)_{t=1, \dots, \theta-1}$ chosen by a benevolent domestic social planner who maximizes the welfare of the representative

consumer taking the global economic conditions $(r_t)_{t=1,\dots,\theta-1}$ and $(C_{Ft}^W)_{t=1,\dots,\theta}$ as given. These policies will be called *domestically optimal*.

In line with most of the literature we assume that the social planner cannot commit to her future policies and look for time-consistent optimal policy paths. In the real world, policymakers can find ways to constraint their use of certain policies, for example membership to the World Trade Organization for trade, or membership of the OECD or the European Union precludes for capital controls. Thus we assume that the social planner may be prevented from using certain policy instruments. A key contribution of our analysis is to determine the benefits of committing to not use certain policy instruments. More formally, we assume that the social planner of country j uses policy instruments in a subset of the possible instruments $\{i, \tau^m, \tau^b\}$. The interest rate i_j is always part of the usable instruments (subject to the zero lower bound) but the set of available policy instruments may be otherwise restricted. We denote by n_{jt} the values of country j .

One feature of the model that makes it tractable is that the determination of the optimal time-consistent policy can be decomposed in a sequence of independent period- t problems. To see this, observe that iterating forward on (1) and (14) and using $U_\theta = \bar{L} + B_\theta$ gives,

$$U_{jt} = e^{-\bar{\rho}_{jt}} \left[\bar{L} + e^{\bar{r}_t} B_{jt} + \sum_{k=t}^{\theta-1} e^{\bar{\rho}_{jk}} V_{jk} \right]$$

where $\bar{\rho}_{jt} \equiv \sum_{k=t}^{\theta} \rho_{jk}$, $\bar{r}_t \equiv \sum_{k=t}^{\theta} r_k$ and

$$V_{jt} \equiv u(C_{jt}) + e^{\bar{r}_t - \bar{\rho}_{jt}} X_{jt}, \quad (21)$$

is the flow utility resulting from the period- t levels of consumption and trade balance. This flow utility increases with current consumption and with the trade balance, which is accumulated in foreign assets and consumed in the final period. The social planner's problem, thus, can be decomposed in a sequence of period- t problems where the social planner maximizes V_{jt} over the period- t policy variables. In each period- t problem the social planner maximizes V_{jt} subject to the full employment constraint $L_{jt} \leq \bar{L}$ and taking the future policy variables as given. The impact of current policies on consumption and the trade balance is given by equations (15), (18) and (19). Note that the optimal policy does not depend on the beginning-of-period level of foreign assets B_{jt} , which is a determinant of welfare but not a relevant state variable for the problem of determining the optimal policy instruments.

We leave the characterization of optimal policies for the next section. For now we simply note that if policy is optimal, unemployment can arise only when monetary policy is constrained by the zero lower bound.

Proposition 4 *Assume that the policy of country j is determined by a time-consistent social planner who maximizes domestic welfare. Then in any period t either there is full employment or the economy is in a liquidity trap,*

$$\forall t, L_{jt} = \bar{L} \text{ or } i_{jt} = 0.$$

Proof. If $L_{jt} < \bar{L}$, lowering the interest rate raises consumption C_{jt} by equation (19). If $\sigma < 1$ this also increases the trade balance X_{jt} and by equation (21), welfare V_{jt} . Hence the policy cannot be optimal if $L_{jt} < \bar{L}$ and $i_{jt} > 0$. ■

4 International policy coordination

In the remainder of the paper we look at equilibria of the global economy in which national social planners coordinate their policies or not. We compare how the international policy spillovers and the benefits of international coordination depend on the policy instruments. The first section formally defines the equilibria without and without cooperation. Section 4.2 characterizes the international spillovers associated with the different policies. Section 4.3 investigates the case where the national social planners can use only monetary policy (possibly including the inflation target) and we then introduce the other instruments one by one: tariffs in section 4.4 and capital controls in section 4.5.

4.1 Equilibria

We consider symmetric equilibria in which all countries use the same set of policy instruments and have the same inflation target π^* and the same time-varying discount rate,

$$\forall j, t \rho_{jt} = \rho_t. \tag{22}$$

Asymmetric equilibria in which countries may have different discount rates or use different policy instruments will be analyzed in section ??.

There are two global market clearing conditions. In any period t the countries' trade balances and net foreign assets sum up to zero,

$$\int X_{jt}dj = \int B_{jt}dj = 0, \quad (23)$$

and global imports are the sum of imports across all countries,

$$C_{Ft}^W = \alpha_F \int (S_{jt}^m)^{\alpha_H} C_{jt}dj. \quad (24)$$

These global markets clearing conditions and integrating $X_{jt} = (S_{jt})^{1-\gamma} C_{Ft}^W - \alpha_F (S_{jt}^m)^{\alpha_H} C_{jt}$ over all countries j imply

$$\int S_{jt}^{1-\gamma} dj = 1. \quad (25)$$

This equation captures the fact that terms of trade are relative prices that cannot move in the same direction in all countries. Changing the terms of trade of a given country changes the terms of trade of the rest of the world in the opposite direction.

We look at a Nash equilibrium between national social planners (NSP) who do not coordinate. A Nash equilibrium is composed of (i) global economic conditions $(r_t)_{t=1,\dots,\theta-1}$ and $(C_{Ft}^W)_{t=1,\dots,\theta}$; (ii) policies $(i_{jt}, \tau_{jt}^m, \tau_{jt}^b)_{j,t=1,\dots,\theta-1}$ and allocations $(C_{Hjt}, C_{Fjt}, C_{jt}, L_{jt}, \Pi_{jt})_{t=1,\dots,\theta}$ for all countries $j \in [0, 1]$ such that:

- the policy of country j is time-consistent and maximizes domestic welfare given the global economic conditions,
- country allocations satisfy the equilibrium conditions given country policies and global economic conditions;
- the global markets clearing conditions (23) and (24) are satisfied.

We compare this Nash equilibrium to the equilibrium where national policies are all set by a global social planner (GSP) who maximizes the welfare of the representative country. The GSP equilibrium can also be interpreted as the result of international coordination between the national social planners. The NSP and GSP equilibria are characterized and compared in the following sections.

4.2 Policy spillovers

The welfare implications of policy coordination depends on the policy spillovers. The spillovers are measured as follows. Consider the nominal interest rate. Assume that a small group of countries j of mass ε change their interest rate i_j . The mass ε is small and we will leave out the second-order terms in that variable. We start from a symmetric equilibrium in which all countries have the same level of employment $L < \bar{L}$, interest rate i , inflation target π and terms of trade $S = 1$. We estimate the impact of marginal change in i_j on the welfare of the countries that change their inflation target, U_j , on welfare in the rest of the world, U_{-j} , as well as global welfare,

$$U^W = \varepsilon U_j + (1 - \varepsilon) U_{-j}.$$

The welfare impact of changing the inflation target is given by⁷

$$\frac{1}{C} \frac{\partial U_j}{\partial i_j} = -(\eta + \alpha_H \sigma), \quad (26)$$

$$\frac{1}{C} \frac{\partial U_{-j}}{\partial i_j} = \varepsilon(\eta - \alpha_F \sigma), \quad (27)$$

$$\frac{1}{C} \frac{\partial U^W}{\partial i_j} = -\varepsilon \sigma, \quad (28)$$

where $\eta = \alpha_F [\gamma - 1 + \alpha_H (1 - \sigma)]$ is the semi-elasticity of the trade deficit with respect to the interest rate (see Table 1). Increasing the inflation target in a subset of countries has a welfare switching as well as a welfare-augmenting effect. The welfare-switching effect comes from the impact on trade balances: it is captured by the terms in η in expressions (26) and (27), which cancel out in aggregate. The welfare-augmenting effect comes from the positive impact of raising the inflation target on demand, both at home (term $\alpha_H \sigma$) and in the rest of the world (term $\alpha_F \sigma$).

One can compute the spillovers for the other policy instruments τ_j^m and τ_j^b in a similar way. The spillovers for all the instruments are summarized in the following table.

Table 2. International spillovers

⁷This assumes $\bar{r} = \bar{p}$. These expressions are derived in Appendix B.

	i_j	τ_j^m	τ_j^b
U_j	$-\eta - \alpha_H \sigma$	$\alpha_H \alpha_F (1 - \sigma)$	$\eta - \alpha_F \alpha_F \sigma \tau^b$
U_{-j}	$\eta - \alpha_F \sigma$	$-\alpha_F (\alpha_H + \alpha_F \sigma)$	$-\eta + \alpha_F \alpha_F \sigma \tau^b$
U^W	$-\sigma$	$-\alpha_F \sigma$	0

The tariff increases the welfare of the country that imposes but decreases the welfare of the rest of the world. The impact on total welfare is negative, i.e., a tariff war is a negative sum game. This is because tariffs decrease global demand and global employment.

We already know from Table 1 that domestic welfare increases with the tax on inflows. However, the welfare of the rest of the world is decreased and total welfare is unchanged. A capital war is a zero-sum game. This result is intuitive because each country attempts to gain from reducing its terms of trade in a capital war, but in a symmetric allocation the terms of trade must remain equal to 1.

4.3 Monetary wars

As a benchmark we look at conventional monetary wars where the instruments are limited to the nominal interest rate i ($\tau^m = \tau^b = 0$). All the countries have the same inflation target π^* , which is taken as given. In a second step we consider unconventional monetary wars where the inflation target π^* is also part of the instruments.

In a symmetric Nash equilibrium the terms of trade are equal to one in all periods. This, together with equation (18) and $\tau_t^b = 0$ implies,

$$r_t = i_t - \pi_{t+1}.$$

The Nash equilibrium can be constructed by iterating backwards as follows. In the last period θ there is full employment and inflation is equal to the target in all countries. In the previous period $\theta - 1$, using (19) with $\tau^b = \tau^m = 0$ and $r_\theta = i_\theta - \pi^*$ gives log consumption for the representative country

$$c_{\theta-1} = -\sigma (i_{\theta-1} - \pi^* - \rho_{\theta-1}).$$

We know from Proposition 4 that the global economy is either at full employment or in a liquidity trap. If there is full employment then $c_{\theta-1} = 0$ since $C_{\theta-1} = \bar{L} = 1$. This is consistent with the zero lower bound if and

only if $\rho_{\theta-1} + \pi^* \geq 0$. If this condition is violated, there is less than full employment and the nominal interest rate is equal to zero. In general the nominal interest rate is given by

$$i_{\theta-1} = (\rho_{\theta-1} + \pi^*)^+ .$$

The Nash equilibrium can be derived by continuing these iterations backwards. There is full employment in all periods if and only if the condition $\rho_t + \pi^* \geq 0$ is satisfied for all t . The global economy is in a liquidity trap with less than full employment in any period in which this condition is violated. Note that a liquidity trap in period t lowers inflation to $\underline{\pi}$ so that the condition for a liquidity trap in the previous period is weaker, $\rho_{t-1} + \underline{\pi} < 0$. That is, the expectation of a liquidity trap tends to pull the economy into a liquidity trap.⁸

It is easy to see that a time-consistent Global Social Planner chooses the same policies as uncoordinated national social planners. The GSP sets the nominal interest rate so as to maximize the welfare of the representative country. The optimal GSP policy is always to achieve full employment if possible and if not, to maximize demand and employment by setting the nominal interest rate to zero. Thus there are no gains from international coordination in a conventional monetary war.

Our results so far are summarized in the following Proposition.

Proposition 5 *Assume that the only policy instrument available to national social planners is the nominal interest rate. Then in the Nash equilibrium between national planners there is full employment in all periods if and only if $\rho_t + \pi^* \geq 0$ for all t . The global economy falls in a liquidity trap with the same level of unemployment in all countries in any period t that violates this condition. There is no gain from international policy coordination.*

Proof. See discussion above. ■

The Nash equilibrium is no longer symmetric if countries have different inflation targets π_j^* . In a global liquidity trap the countries with higher inflation targets can depreciate their currencies and increase their employment and welfare above those in countries with lower inflation targets. There is still no Pareto gain from international policy coordination.

⁸This does not lead to self-fulfilling traps because the model has a final period in which it is not in a liquidity trap.

This raises the question of the equilibrium when each country can choose its inflation target. To address this question let us assume that each country j sets its inflation target π_j^* before period 1 (say in a new period 0). The Nash equilibrium from period 1 onwards is then determined as before conditional on the inflation targets. In the period-0 Nash equilibrium each country sets its inflation target so as to maximize domestic welfare taking the other countries' inflation target as given. Then we have the following result.

Proposition 6 *Assume that the national social planners can choose their inflation targets before period 1. Then in a symmetric Nash equilibrium social planners set an inflation target $\pi^* \geq -\min \rho_t$ and $i_{jt} = \rho_t + \pi^*$. There is full employment in all countries. There is no benefit from international coordination.*

Proof. There cannot be unemployment in a symmetric Nash equilibrium, otherwise any social planner could increase domestic welfare by raising the domestic inflation target π_j^* . Hence social planners set an inflation target (it must be the same in all countries in a symmetric equilibrium) such that $\rho_t + \pi^* \geq 0$ for all t or $\pi^* \geq -\min \rho_t$. The inflation target is indeterminate as long as it satisfies this condition. A global social planner also increases the inflation target to any level satisfying this condition to maximize the welfare of the representative country. ■

4.4 Trade wars

In a trade war we assume that the two available policy instruments are the nominal interest rate i and the tax on imports τ^m . In a symmetric allocation with unemployment an individual country increases its own employment and welfare by increasing its tariff on imports, as we know from Table 1. One question is how welfare and employment are affected when all countries use a tariff on imports.

As shown by the following result

Proposition 7 *Consider a symmetric Nash equilibrium in which all national social planners use the tax on imports τ^m in addition to the nominal interest rate i . Then in any period t the global economy is in one of the following two regimes:*

(i) there is full employment in all countries and the gross tax on imports is set in all countries at a level $T_t^m = \underline{T}^m > 1$ satisfying

$$\underline{T}^m = \alpha_H + \frac{\alpha_F}{\sigma} + \gamma\alpha_F \left(\frac{\alpha_F}{\alpha_H} + \frac{1}{\sigma} \right) \left(\frac{1}{\underline{T}^m} - 1 \right), \quad (29)$$

(ii) all countries are in a liquidity trap with less than full employment, and set the gross tax on imports at

$$\bar{T}^m = \alpha_F + \frac{\alpha_H}{\sigma}. \quad (30)$$

The equilibrium tariff is larger if there is unemployment

$$\bar{T}^m > \underline{T}^m.$$

The global social planner sets the trade taxes to zero ($T^m = 1$).

Proof. See Appendix ■

There are two possible regimes for the global economy. If global demand is sufficiently high there is full employment but all countries impose a tariff in the Nash equilibrium. Each country tries to manipulate the terms of trade in its favor, like in the textbook static tariff war. The tariff restricts the supply of home good to the rest of the world and raises its price in terms of foreign goods. However the tariff represses consumption in the first period, so that a fall in the interest rate is required to maintain full employment. The tariff given by (29) is decreasing in the micro-elasticity of substitution, γ , and decreasing in the elasticity of intertemporal substitution σ .

If demand is low the economy is in a liquidity trap and there is less than full employment. Countries now tax imports in order to raise employment rather than increase their terms of trade (given $i = 0$ the terms of trade are not affected by the tariff). The equilibrium tariff does not depend on the microelasticity γ because the tariff does not affect exports in a liquidity trap. The marginal impact of the tariff on employment and welfare decreases with σ (see Table 1), which explains why the equilibrium tariff is also decreasing in σ . A tariff war decreases employment when the economy is in a global liquidity trap.

The Nash equilibrium can be constructed by iterating backwards from period θ . Consider the equilibrium in period $t = \theta - 1$, or in any period

such that there is full employment in $t + 1$. There is an equilibrium with full employment if and only if

$$\rho_t + \pi^* \geq \alpha_F \tau_t^m.$$

The level of τ_t^m in turn depends on whether there is full employment or not, leading to multiple equilibria. If

$$\alpha_F \underline{\tau}^m \leq \rho_t + \pi^* \leq \alpha_F \bar{\tau}^m, \quad (31)$$

then there are two equilibria in period t : one equilibrium with full employment and a low tariff $\underline{\tau}^m$ and one equilibrium with less than full employment and a high tariff $\bar{\tau}^m$. The multiplicity comes from the fact that higher tariffs, by lowering demand, pull the economy into a global liquidity trap where all countries have incentives to raise their tariffs to boost their employment. If $\rho_t + \pi^* > \alpha_F \bar{\tau}^m$ there is one unique equilibrium with full employment and if $\rho_t + \pi^* < \alpha_F \underline{\tau}^m$ there is one unique equilibrium with a global liquidity trap.

It is very easy for condition (31) to be satisfied under plausible calibrations of the model. Figure [.] shows the variations of the high and low tariffs with σ . The tariff is much higher under unemployment: hot vs. cold tariff war. For $\sigma = 0.5$ (our benchmark calibration), the low and high tariff rates are respectively $\underline{\tau}^m = 9.8\%$ and $\bar{\tau}^m = 47.0\%$. With $\alpha_F = 0.4$ and $\pi^* = 2\%$ the risk of self-fulfilling tariff war arises if ρ_t is below 18.8%, which certainly the case under any plausible calibration. Thus, the risk of self-fulfilling trade war becomes chronic once national social planners are allowed to use tariffs.

The welfare impact of a trade war depends a lot on whether the global economy is in a liquidity trap. Figure 1 shows how the welfare loss of a trade war varies with ρ under our benchmark calibration with $\pi^* = 2\%$ and $\theta = 1$. It is assumed that economy is in the best possible equilibrium. If ρ exceeds $\alpha_F \bar{\tau}^m - \pi^* = 1.7\%$, there is full employment and the welfare loss, which comes purely from the distortion of production and consumption towards the home good, is relatively small, equivalent to a 0.1% reduction in period-1 consumption. The welfare loss from a trade war is much larger when ρ falls below 1.7%. For ρ between -2% and 1.7% the tariff war pulls the economy in a global liquidity trap that would not exist without tariff war. Once the global economy has switched to the regime with unemployment, the tariff increases to $\bar{\tau}^m = 47\%$, which further increases unemployment. The impact of the trade war on unemployment is large, as shown by Figure 2. The trade war increases the unemployment rate by about 6.5 percent. This

amplification mechanism leads to a discontinuity in the welfare loss. The welfare loss stabilizes at 9.0% of period-1 consumption when ρ is lower than -2% (i.e., when unemployment arises even if there is no trade war).

Figure 3 shows how the dynamics of unemployment are affected by a trade war. The figure is based on the following parameter values: $\pi = 2\%$, $\theta = 4$, $\rho_t = -3\%$ for $t < 4$. The line labeled "transitory trade war" corresponds to the assumption that countries apply a tariff only if there is unemployment, whereas the line labeled "permanent trade war" assumes that countries use tariffs in all periods. The impact of a trade war on unemployment is large. Unemployment is higher in a transitory trade war because the intertemporal taxation of consumption is more distortive.

4.5 Capital wars

In a capital war the two available policy instruments are the nominal interest rate i and the tax on capital inflows τ^b . We look at a two-period economy ($\theta = 2$). The equilibria can then be characterized as follows.

Proposition 8 *Assume that the policy instruments available to national social planners are the nominal interest rate and the tax on capital inflows. Then in a two-period economy ($\theta = 2$)*

(i) *If $\rho \geq -\pi^*$, all countries set $i = \rho + \pi^*$ and*

$$T^b = 1 - \frac{1}{\gamma}. \quad (32)$$

There is full employment and welfare is the same as if countries did not use capital flow taxes.

(ii) *If $\rho < -\pi^*$ and $\gamma < 2$, all countries set $i = 0$ and*

$$T^b = \frac{\gamma - \alpha_F(1 - \sigma)}{\sigma}. \quad (33)$$

There is less than full employment and welfare is the same as if countries did not use capital flow taxes.

(iii) *If $\rho < -\pi^*$ and $\gamma \geq 2$ there is no symmetric Nash equilibrium.*

(iv) *There is no welfare gain from international policy coordination.*

Proof. See Appendix. ■

Like in the case of a trade war, the equilibrium policies crucially depend on whether the global economy is in a liquidity trap or not. Under full employment countries attempt to manipulate the terms of trade in their favor by subsidizing capital inflows and appreciating their currencies. The result in equilibrium is an increase in the interest rate that offsets the impact of the capital inflow subsidy. National social planners internalize the fact that the interest rate on external borrowing is higher. In equilibrium this interest rate must be high enough to remove any incentives to engineer a marginal appreciation.

In a global liquidity trap countries attempt to manipulate their terms of trade in the other direction. They try to increase employment by depreciating their currency, which leads to a fall in the real interest rate. In a symmetric equilibrium the interest rate must fall by enough to discourage any marginal accumulation of foreign assets.

As symmetric Nash equilibrium exists, when the zero lower bound is binding, only if the microelasticity γ is smaller than 2. The case $\gamma \geq 2$ is relevant given the existing estimates of the micro-elasticity of substitution in the literature. In this case there is no symmetric Nash equilibrium because country welfare becomes a convex function of the tax rate on inflows at the extremum given by (33). Countries strictly benefit from deviating from the Nash equilibrium, either by depreciating their currency and reaching full employment or by letting capital flow in to finance a large level of consumption.

The global economy endogenously divides itself into two groups of countries: a group of countries with a more competitive currency, a trade surplus, and full employment, and a group of countries with a less competitive currency, a trade deficit and some unemployment. These countries complain about different things: the deficit countries about their unemployment and the surplus countries about the low return that they received on the fruit of their labor. The cost of one country is the benefit of another: the full employment of surplus countries is made possible by the unemployment in deficit countries, and the deficit countries can find consolation for their lower income in their ability to borrow abroad at a low interest rate. In equilibrium the welfare of surplus countries and deficit countries is the same.

5 Conclusions

The paper opens several directions for further research. Making the model less symmetric would allow us to look at questions that have not been analyzed in this paper. For example, assuming that countries differ in their time preferences (the discount rates ρ) would make it possible to examine how a "global savings glut" in one part of the world may affect the benefits of international policy coordination. Another relevant source of asymmetry is if countries have access to different policy instruments. In the real world some countries are committed not to use certain policy instruments. For example, OECD and EU membership preclude the use of capital controls except in exceptional circumstances. WTO membership also puts restrictions on trade policies (although the limits of these restrictions are increasingly tested). One could also assume that countries have different sizes or home bias.

Another question is the robustness of trigger strategy equilibria in which free trade is supported by the threat of a trade war. It would be interesting to know, in this context, whether a trade war is made more or less likely by a fall in global demand leading to unemployment.

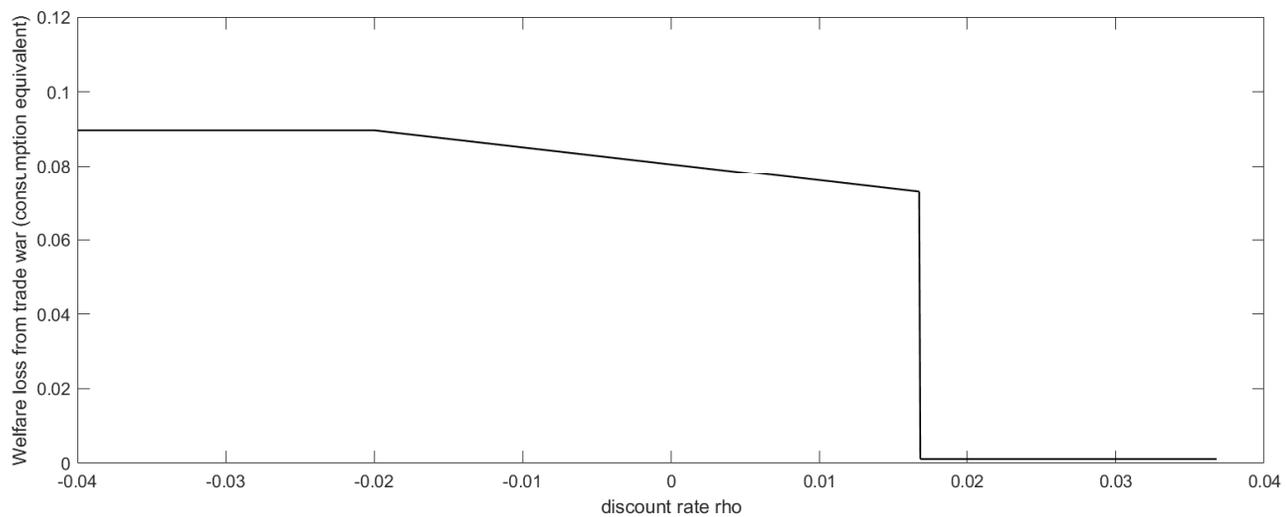


Figure 1: Variation of welfare loss from trade war (as a share of period-1 consumption) with discount rate ρ

u

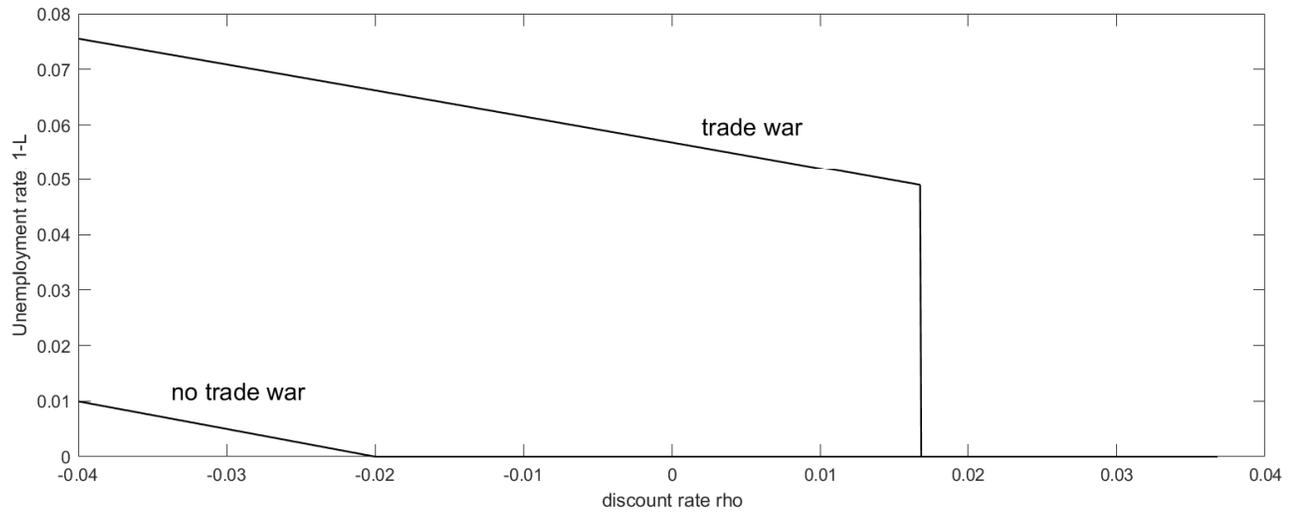


Figure 2: Variation of unemployment rate with discount rate ρ

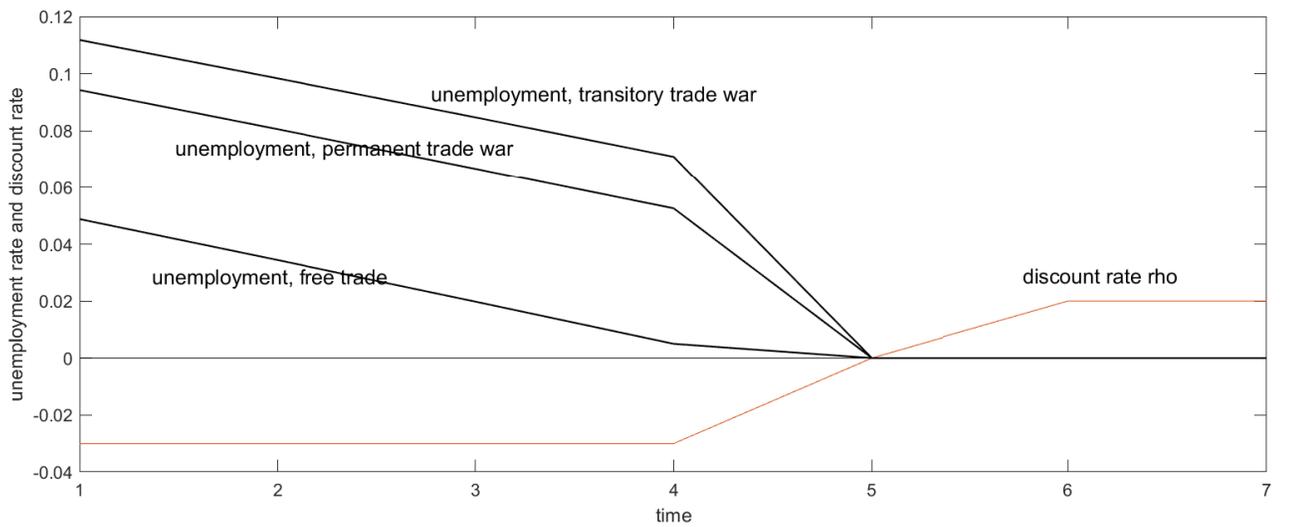


Figure 3: Unemployment rate in a dynamic trade war

APPENDICES

APPENDIX A. MODEL WITH MONEY AND NOMINAL BONDS

This appendix derives the first-order conditions in the model with money and nominal bonds. We assume that the representative consumer derives utility $v(M_t/W_t)$ from real money balances. The demand for money is satiated when real money balances reach a level \bar{m} , that is, $v'(m) > 0$ for $m < \bar{m}$ and $v'(m) = 0$ for $m \geq \bar{m}$.

For any period $t < \theta$ the Bellman equation for the representative consumer is

$$U_t(B_t, B_t^n, M_{t-1}) = \max_{C_{Ht}, C_{Ft}, B_{t+1}, B_{t+1}^n, M_t} u(C_t) + v\left(\frac{M_t}{W_t}\right) + e^{-\rho t} U_{t+1}(B_{t+1}, B_{t+1}^n, M_t)$$

subject to (2) and the budget constraint

$$P_t \frac{B_{t+1}}{R_t T_t^b} + \frac{B_{t+1}^n}{I_t} + M_t + W_t C_{Ht} + T_t^m P_t C_{Ft} = W_t L_t + Z_t + P_t B_t + B_t^n + M_{t-1},$$

where $I_t = \exp(i_t)$ is the gross nominal interest rate and B_t^n is the consumer's holdings of nominal bonds denominated in the domestic currency. Nominal bonds are traded domestically only.

In the final period θ the problem becomes

$$\begin{aligned} U_\theta(B_\theta, B_\theta^n, M_{\theta-1}) &= \max C_{H\theta} + C_{F\theta} + v\left(\frac{M_\theta}{W_\theta}\right) \\ \text{s.t. } M_\theta + W_\theta C_{H\theta} + P_\theta C_{F\theta} &= W_\theta L_\theta + Z_\theta + P_\theta B_\theta + B_\theta^n + M_{\theta-1}. \end{aligned}$$

For any period $t < \theta$ arbitrage between real and nominal bonds implies that the gross nominal interest rate $I_t = \exp(i_t)$ satisfies

$$I_t = R_t T_t^b \frac{P_{t+1}}{P_t}.$$

Using $S_t = W_t/P_t$ and $\Pi_{t+1} = W_{t+1}/W_t$ and taking the log gives

$$s_t = i_t - \pi_{t+1} - r_t - \tau_t^b + s_{t+1}.$$

Iterating forward on this equation and using $s_\theta = 0$ (because of the linearity of the final period utility, which implies $S_\theta = W_\theta/P_\theta = 1$) then gives equation (18) in the text.

The Euler equation can be written

$$u'(C_t)(S_t^m)^{\alpha_F} = e^{-\rho_t} \frac{I_t}{\Pi_{t+1}} u'(C_{t+1})(S_{t+1}^m)^{\alpha_F} \quad (34)$$

for any period $t < \theta - 1$ and

$$u'(C_{\theta-1})(S_{\theta-1}^m)^{\alpha_F} = e^{-\rho_{\theta-1}} \frac{I_{\theta-1}}{\Pi_\theta} \quad (35)$$

in period $\theta - 1$. These equations characterize the intertemporal substitution of home good consumption. Note that $(S_t^m)^{\alpha_F}$ is the onshore price of the home good in terms of home consumption so that $u'(C_t)(S_t^m)^{\alpha_F}$ is the marginal utility gain from consuming an extra unit of home good. Taking the log of these equations

$$\begin{aligned} c_t &= -\sigma(i_t - \pi_{t+1} - \rho_t) - \sigma\alpha_F [(s_{t+1} - s_t) - (\tau_{t+1}^m - \tau_t^m)] + c_{t+1}, \\ c_{\theta-1} &= -\sigma(i_{\theta-1} - \pi_\theta - \rho_{\theta-1}) + \sigma\alpha_F s_{\theta-1}. \end{aligned}$$

Iterating forward on the expression for c_t gives

$$c_t = -\sigma \sum_{t'=t}^{\theta-1} (i_{t'} - \pi_{t'+1} - \rho_{t'}) + \sigma\alpha_F (s_t - \tau_t^m).$$

Using (18) to substitute out s_t then gives equation (19) in the text.

The first-order conditions for money demand are,

$$v' \left(\frac{M_t}{W_t} \right) = u'(C_t)(S_t^m)^{\alpha_F} (1 - e^{-i_t}) \text{ for } t < \theta, \quad (36)$$

$$v' \left(\frac{M_\theta}{W_\theta} \right) = 1. \quad (37)$$

Equation (37) shows that in period θ it is always possible for the social planner to set M_θ to a level that ensures $\pi_\theta = \pi^*$, consistent with full employment in that period.

We now show by backward iteration that in any period $t < \theta$ there is a unique level of money supply M_t that ensures that inflation is equal to the target, $\pi_t = \pi^*$ if there is full employment.

In any period $t < \theta$ in which the zero-bound is not binding the social planner can implement the inflation target by setting money supply M_t at the appropriate level. To see this consider a policy path with $i_t > 0$ that pins down C_t , S_t and S_t^m . Then all the variables on the r.h.s. of (36) being fixed, P_t and so W_t (since S_t is fixed) increases proportionately with M_t . It is thus possible for the social planner to set M_t so as to reach the inflation target $\pi_t = \pi^*$. This is not true if $i_t = 0$ because then equation (36) only implies $P_t \leq M_t/\bar{m}$ and raising money supply does not raise the price level.

Show that the nominal interest rate path and inflation target can be implemented with money. Show that if foreigners can buy nominal bonds and money the tax T^b must also apply to these assets.

APPENDIX B. COMPARATIVE STATICS

The semi-elasticities are reported in Table B1. The expressions for S_{jt} and C_{jt} directly follow from (18) and (19). The other expressions follow from

$$\begin{aligned} L_{jt} &= C_{Hjt} + S_{jt}^{-\gamma} C_{Ft}^W, \\ X_{jt} &= S_{jt}^{1-\gamma} C_{Ft}^W - C_{Fjt} \\ V_{jt} &= u(C_{jt}) + e^{r(t,\theta) - \rho(t,\theta)} X_{jt}, \end{aligned}$$

where

$$C_{Hj} = \alpha_H \exp[-\alpha_F (s_t - \tau_t^m)] C_t, \quad (38)$$

$$C_{Fj} = \alpha_F \exp[\alpha_H (s_t - \tau_t^m)] C_t. \quad (39)$$

Table B 1. Semi-elasticities of macroeconomic variables and welfare with respect to policy instruments

	i_{jt}	τ_{jt}^m	τ_{jt}^b
S_{jt}	1	0	-1
C_{jt}	$-\alpha_H \sigma$	$-\alpha_F \sigma$	$-\alpha_F \sigma$
L_{jt}	$-\alpha_F \gamma - \alpha_H (\alpha_F + \alpha_H \sigma)$	$\alpha_H \alpha_F (1 - \sigma)$	$\alpha_F [\gamma + \alpha_H (1 - \sigma)]$
X_{jt}	$-\alpha_F [\gamma - 1 + \alpha_H (1 - \sigma)]$	$\alpha_F (\alpha_H + \alpha_F \sigma)$	$\alpha_F (\gamma - 1 + \alpha_H + \alpha_F \sigma)$
U_{jt}	$-\alpha_H \sigma - \alpha_F [\gamma - 1 + \alpha_H (1 - \sigma)]$	$\alpha_H \alpha_F (1 - \sigma)$	$\alpha_F [\gamma - 1 + \alpha_H (1 - \sigma)]$

Consider a symmetric equilibrium with $\tau_j^m = \tau_j^b = 0$, $S_j = 1$, and $C_{Fj} = C_F^W = \alpha_F C_j$. The semi-elasticities with respect to the policy instruments reported in Table 1 for S_j , C_j , L_j and B_j easily follow from the expressions above. The semi-elasticity for U_j can be derived by noting that $i_j - \pi_j = r$ so that by equation (34), $u'(C_j) = e^{r-\rho}$. Hence,

$$\frac{e^{\rho-r}}{C} \frac{\partial V_j}{\partial n_j} = \frac{1}{C} \frac{\partial C_j}{\partial n_j} + \frac{1}{C} \frac{\partial B_j}{\partial n_j}.$$

APPENDIX C. PROOFS

Proof of Proposition 2

We show how at most one allocation can be derived from a policy $(i_{jt}, \tau_{jt}^m, \tau_{jt}^x, B_{jt})_{t=1, \dots, \theta-1}$. We proceed by iteration from period θ backwards. In period θ there is full employment and inflation is equal to the target ($L_{j\theta} = \bar{L}$, $\Pi_{j\theta} = \Pi_j^*$). By equation (9) with $W_{j\theta} = P_{j\theta}$ and $T_{j\theta}^x = 1$, the home consumption of home good is $C_{Hj\theta} = \bar{L} - C_{F\theta}^W$ and by equation (7) the home consumption of foreign good is $C_{Hj\theta} = C_{F\theta}^W + B_{j\theta}$.

In period $\theta-1$ the trade balance is $X_{j\theta-1} = B_{j\theta}/R_\theta - B_{j\theta}$. One can derive $C_{j\theta-1}$ from equation (35). Using this expression to substitute out $C_{j\theta-1}$ in equation (15) then gives

$$X_{j\theta-1} = (S_{j\theta-1}^x)^{1-\gamma} C_{F\theta-1}^W - \alpha_F \left(e^{-\rho_{j\theta-1}} \frac{I_{j\theta-1}}{\Pi_{j\theta}} \right)^{-\sigma} (S_{j\theta-1}^m)^{\alpha_H + \alpha_F \sigma}.$$

The r.h.s. of this equation strictly decreases with $S_{j\theta-1}$ given $\tau_{j\theta}^x$ and $\tau_{j\theta}^m$ so that the equation has at most one solution $S_{j\theta-1}$. The values of $C_{Hj\theta-1}$ and $C_{Fj\theta-1}$ can then be derived from (11) and (12). Employment $L_{j\theta-1}$ is derived from (13). The policy is admissible if $L_{j\theta-1} \leq \bar{L}$. Inflation $\Pi_{j\theta-1}$ then results from (16) and (17). The iterations for previous periods are similar.

Proof of Proposition 3

Given the targeting rule (16), the allocation $(C_{Hjt}, C_{Fjt}, L_{jt}, \Pi_{jt})_{t=1, \dots, \theta}$ is entirely determined by the terms of trade relevant for exports and imports, $s_{jt}^x = s_{jt} + \tau_{jt}^x$ and $s_{jt}^m = s_{jt} - \tau_{jt}^m$. Let us denote with tilde the policy and corresponding terms of trade that yield the same allocation as the original policy but with a zero tax on exports. Thus we have $s_{jt}^x = \tilde{s}_{jt}^x$ and $s_{jt}^m = \tilde{s}_{jt}^m$

for $t = 1, \dots, \theta - 1$. Since $\tilde{\tau}_{jt}^x = 0$, $\tilde{s}_{jt}^x = \tilde{s}_{jt} = s_{jt}^x = s_{jt} + \tau_{jt}^x$. Using $\tilde{s}_{jt} = s_{jt} + \tau_{jt}^x$ and $s_{jt}^m = s_{jt} - \tau_{jt}^m = \tilde{s}_{jt}^m = \tilde{s}_{jt} - \tilde{\tau}_{jt}^m$ then implies $\tilde{\tau}_{jt}^m = \tau_{jt}^m + \tau_{jt}^x$. Equation (18) and $\tilde{s}_{jt} = s_{jt}$ imply

$$\sum_{k=t}^{\theta-1} (\tilde{i}_k - \tilde{\tau}_k^b) = \sum_{k=t}^{\theta-1} (i_k - \tau_k^b) + \tau_t^x. \quad (40)$$

The equality $\tilde{c}_{jt} = c_{jt}$, equation (19) and using (40) to substitute out $\sum_{k=t}^{\theta-1} \tilde{i}_k$ then imply

$$\forall t, \quad \sum_{k=t}^{\theta-1} \tilde{\tau}_k^b + \tau_t^x = \sum_{k=t}^{\theta-1} \tau_k^b.$$

It follows that $\tilde{\tau}_{jt}^b = \tau_{jt}^b + \tau_{jt+1}^x - \tau_{jt}^x$ and $\tilde{i}_{jt} = i_{jt}$, as stated in the Proposition.

Proof of Proposition 7

In period t the social planner of country j maximizes domestic welfare over the policy instruments i_{jt} and τ_{jt}^m subject to $L_{jt} \leq \bar{L}$. Dropping the time index t , the Lagrangian for this problem is

$$\mathcal{L} = u(C_j) + e^{\bar{r}-\bar{p}} (S_j^{1-\gamma} C_F^W - C_{Fj}) + \lambda e^{\bar{r}-\bar{p}} (\bar{L} - S_j^{-\gamma} C_F^W - C_{Hj}).$$

We look for a symmetric Nash equilibrium in which all countries have the same consumption levels C_H , C_F and $C_F^W = C_F$. Using equations (??)-(??) and $u'(C_j) C_j = e^{\bar{r}-\bar{p}} C_{Hj} / \alpha_H$, the first-order conditions for i_j and τ_j^m can be written

$$\begin{aligned} (\alpha_F + \alpha_H \sigma) (\lambda C_H + C_F) &= \sigma C_H + \gamma (1 - \lambda) C_F, \\ (\alpha_H + \alpha_F \sigma) C_F &= \alpha_F \left[\frac{\sigma}{\alpha_H} + \lambda (1 - \sigma) \right] C_H. \end{aligned}$$

Manipulations of the second expression give

$$\begin{aligned} 1 - \lambda &= \frac{\alpha_H + \alpha_F \sigma}{\alpha_H (1 - \sigma)} \left(1 - \frac{\alpha_H C_F}{\alpha_F C_H} \right), \\ \lambda C_H + C_F &= \frac{1}{\alpha_F (1 - \sigma)} \left(-\frac{\alpha_F}{\alpha_H} \sigma C_H + C_F \right). \end{aligned}$$

Using these expressions to substitute out λ from the first-order condition for i_j , and $T^m = \frac{\alpha_F C_H}{\alpha_H C_F}$, gives equation (29).

If $i = 0$ the variation of welfare with the tariff is given by,

$$\begin{aligned}\frac{\partial U_j}{\partial \tau_j^m} &= u'(C_j) \frac{\partial C_j}{\partial \tau_j^m} - e^{r-\rho} \frac{\partial C_{Fj}}{\partial \tau_j^m}, \\ &= e^{\bar{r}-\bar{\rho}} C_{Fj} (\alpha_H + \alpha_F \sigma - \sigma T_j^m),\end{aligned}$$

where equations (??), (39), (34) and $C_{Hj}/C_{Fj} = (\alpha_H/\alpha_F) T_j^m$ are used to derive the second equality. This implies equation (30).

One can see that the value of T_j^m given by (30) is larger than that given by (29).

The global social planner does not apply tariffs since this distorts consumption when there is full employment and otherwise reduces employment. One can show that global employment decreases with the tariff rate if Then global employment decreases in the tariff level if and only if

$$T^m < 1 + \frac{\sigma}{\alpha_H (1 - \sigma)}. \quad (41)$$

This condition is satisfied by the equilibrium tariff whether there is unemployment or not.

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